

Important:

- 1. Answer all questions and problems (No solution = no points).
- 2. Full mark = 20 points as arranged in the above table.
- 3. Give your final answer in the correct units.
- 4. Assume $g = 10 \text{ m/s}^2$.
- 5. Mobiles are **<u>strictly prohibited</u>** during the exam.
- 6. Programmable calculators, which can store equations, are not allowed.
- 7. Cheating incidents will be processed according to the university rules.

GOOD LUCK

Part I: Short Problems (2 points each)

SP1. Two vectors \vec{A} and \vec{B} are shown in the figure. Write \vec{A} and \vec{B} in unit vector notation.



SP2. Given the two vectors $\vec{A} = (2\hat{\imath} - 4\hat{j})$ and $\vec{B} = (-2\hat{\imath} + 3\hat{j})$, if $\vec{C} = \vec{A} + 2\vec{B}$, find the angle between \vec{C} and the +x axis.

 $\vec{C} = \vec{A} + 2\vec{B} = (2\hat{\imath} - 4\hat{\jmath}) + 2(-2\hat{\imath} + 3\hat{\jmath}) = -2\hat{\imath} + 2\hat{\jmath}$ $\alpha = tan^{-1} \left(\frac{2}{-2}\right) = -45^{\circ}$ $\theta = 180 - 45 = 135^{\circ} \text{ CCW from +x axis}$

Or

$$\theta_x = \cos^{-1}\left(\frac{C_x}{C}\right) = \cos^{-1}\left(\frac{-2}{2\sqrt{2}}\right) = 135^\circ$$

SP3. The position vector of a particle moving in the xy - plane is given by: $\vec{r} = (2t - t^3)\hat{i} + (4t^2)\hat{j}$ where t is measured in seconds and \vec{r} is measured in meters. Find the speed of the particle at t = 2 seconds.

$$\vec{v} = \frac{d\vec{r}}{dt} = (2 - 3t^2)\hat{\imath} + 8t\hat{\jmath}$$
$$\vec{v}(t=2) = (2 - 3(2)^2)\hat{\imath} + 8(2)\hat{\jmath} = (-10\hat{\imath} + 16\hat{\jmath}) \text{ m/s}$$
$$|\vec{v}(2)| = \sqrt{(-10)^2 + 16^2} \approx 18.9 \text{ m/s}$$

SP4. A ball is thrown **upward** with an initial velocity (v_0) from the top of a building with a height (h). It takes t = 2s to reach its maximum height and then hits the ground at point **B** with a speed of 35 m/s. What is the height of the building (h)?

$$v_{y} = v_{oy} + a_{y}t$$

$$0 = v_{oy} - gt$$

$$v_{oy} = gt = 10(2) = 20 \text{ m/s}$$

$$v_{f}^{2} = v_{i}^{2} - 2g(y_{f} - y_{i})$$

$$(-35)^{2} = (20)^{2} - 2 \times 10(y_{f} - 0)$$

$$y_{f} = -41.25 \text{ m},$$

$$h = 41.25 \text{ m}$$



SP5. The position-time graph of an object moving along the x-axis is shown in the figure. Find the <u>average</u> <u>acceleration</u> between t = 2 s and t = 10 s.

$$a_{av} = \frac{\Delta v}{\Delta t}$$

$$v_2 = \frac{-2 - 6}{12 - 8} = -2 \ m/s$$

$$v_1 = \frac{6 - 2}{4 - 0} = 1 \ m/s$$

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{-2 - 1}{10 - 2} \approx -0.38 \ m/s^2$$



Part II: Long Problems (3 points each)

LP1. Given two vectors: $\vec{A} = (3\hat{\imath} - 4\hat{\jmath} + 4\hat{k})$ and $\vec{B} = (2\hat{\imath} + 3\hat{\jmath} - 7\hat{k})$,

(a) Find $\vec{C} = 2\vec{A} - \vec{B}$ in unit vector notation.

$$\vec{\boldsymbol{C}} = 2\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}} = 2(3\hat{\imath} - 4\hat{\jmath} + 4\hat{k}) - (2\hat{\imath} + 3\hat{\jmath} - 7\hat{k})$$

$$\vec{\boldsymbol{\mathcal{C}}} = 4\hat{\imath} - 11\hat{\jmath} + 15\hat{k}$$

(**b**) Find the vector product $\vec{A} \times \vec{B}$.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 4 \\ 2 & 3 & -7 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = 16\hat{\imath} + 29\hat{\jmath} + 17\hat{k}$$

(c) Find $\vec{C} \cdot (\vec{A} \times \vec{B})$.

$$\vec{\boldsymbol{C}} \cdot (\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}) = 2\vec{\boldsymbol{A}} \cdot (\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}) - \vec{\boldsymbol{B}} \cdot (\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}) = 0 - 0 = 0$$

or

$$\vec{C} \cdot (\vec{A} \times \vec{B}) = 4(16) - 11(29) + 15(17) = 0$$

LP2. An object moves along the **x-axis** with its position as a function of time given by: $x(t) = 4t - 0.125t^4$ where x is measured in *meters* and t is measured in *seconds*.

(a) Find the <u>average velocity</u> between t = 0 s and t = 4 s.

$$v_{av} = \frac{\Delta x}{\Delta t}$$

$$x_2 = 4(4) - 0.125(4)^4 = -16 m$$

$$x_1 = 4(0) - 0.125(0)^4 = 0 m$$

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{-16 - 0}{4 - 0} = -4 \text{ m/s}$$

(b) <u>Find the time</u> when the object <u>changes its direction of motion</u>.

$$v_{x} = \frac{dx}{dt} = 4 - 0.5t^{3}$$
$$v_{x} = 0$$
$$t = 2 \ seconds$$

(c) Find the average speed of the object between $\underline{t = 0}$ and $\underline{t = 4 s}$.

$$s_{av} = \frac{D}{\Delta t}$$

$$D_1 = x(2) - x(0) = 4(2) - 0.125(2)^4 = 8 - 2 = 6 m$$

$$D_2 = |x(4) - x(2)| = |-16 - 6| = 22 m$$

$$s_{av} = \frac{D}{\Delta t} = \frac{6+22}{4-0} = 7 m/s$$

Part III: Questions (Choose the correct answer, one point each)

Q1. Which of the following relations describes the vector \vec{A} in the figure?



Q2. An object is moving along a straight line. Which of the following is true if the object is speeding up?

* v > 0, a < 0* v = 0, a = 0* v < 0, a < 0* v < 0, a > 0

Q3. The value of $\hat{\boldsymbol{\iota}} \cdot (\hat{\boldsymbol{k}} \times \hat{\boldsymbol{j}})$ is

→ -1	* +1	* 0	* Î

Q4. The velocity and acceleration of an object at a certain instant are:

$$\vec{v} = -2\,\hat{j}\,m/s;$$
 $\vec{a} = (3\hat{\imath} + 4\hat{j})\,m/s^2.$

At this instant, the object is

* moving in a straight line and slowing down.

(*) moving in a curved path and slowing down.

- * moving in a straight line and speeding up.
- * moving in a curved path and speeding up.