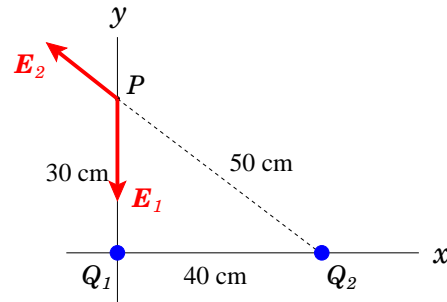


1. Two point charges, $Q_1 = -6.0 \text{ nC}$ and $Q_2 = +8.0 \text{ nC}$, are in the xy -plane as shown. The point P is on the y -axis. Find the x -component and the y -component of the net electric field \vec{E} at P . 6 points



Solution: Since $Q_1 < 0$ and $Q_2 > 0$, the directions of \vec{E}_1 and \vec{E}_2 are as shown. The magnitudes and components of \vec{E}_1 and \vec{E}_2 are

$$E_1 = \frac{k|Q_1|}{0.3^2} = 600.0 \text{ N/C} \implies \begin{cases} E_{1x} = 0.0 \text{ N/C} \\ E_{1y} = -600.0 \text{ N/C} \end{cases}$$

$$E_2 = \frac{k|Q_2|}{0.5^2} = 288.0 \text{ N/C}; \implies \begin{cases} E_{2x} = -E_2 \times \frac{0.4}{0.5} = -230.4 \text{ N/C} \\ E_{2y} = +E_2 \times \frac{0.3}{0.5} = +172.8 \text{ N/C} \end{cases}$$

Then

$$E_x = E_{1x} + E_{2x} = -230.4 \text{ N/C}$$

$$E_y = E_{1y} + E_{2y} = -427.2 \text{ N/C}$$

2. A uniform ring of radius $a = 8 \text{ cm}$ and charge $Q = -12.0 \mu\text{C}$ is fixed in the yz -plane with its centre at the origin O . A point charge $q = +4.0 \mu\text{C}$ is on the x -axis at 12 cm from the origin. Find the net electric field \vec{E} at the point P which is at a distance $x = 6.0 \text{ cm}$ from the origin. 4 points

Solution: Both \vec{E}_{Ring} and \vec{E}_q are in the **negative** x -direction. The magnitudes are

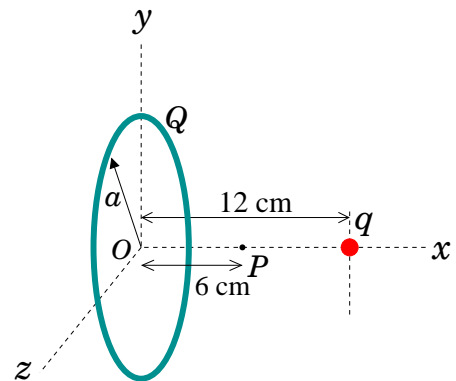
$$E_{Ring} = \frac{k|Q|(0.06)}{(0.06^2 + 0.08^2)^{3/2}} = 6.5 \times 10^6 \text{ N/C}$$

$$E_q = \frac{k|q|}{0.06^2} = 1.0 \times 10^7 \text{ N/C}$$

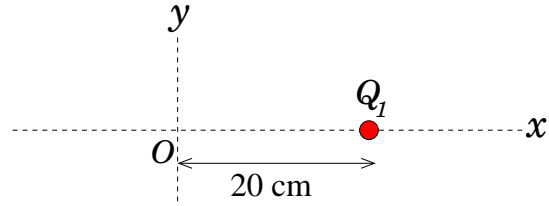
The magnitude of the net electric field is

$$E = E_{Ring} + E_q = 1.65 \times 10^7 \text{ N/C}$$

The direction: in the **negative** x -direction.



3. The point charge $Q_1 = -3.0 \mu\text{C}$ is on the x -axis at a distance of 20 cm from the origin as shown. Find the location of the point charge $Q_2 = +2.0 \mu\text{C}$ on the x -axis such that the net electric field $\vec{E} = 0$ at the origin O . 4 points



Solution: Since Q_1 and Q_2 have opposite signs, and $|Q_2| < |Q_1|$, the point at which $\vec{E} = 0$ (in this case, it is the origin), must lie on the side of Q_2 and not in the middle. Therefore, Q_2 must be **between O and Q_1** . Let this distance be x . Then

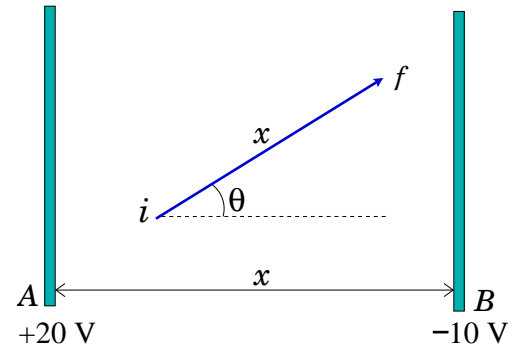
$$\frac{k|Q_1|}{0.2^2} = \frac{k|Q_2|}{x^2} \implies x = 0.16 \text{ m}$$

4. Two charged particles of identical mass $m = 5.0 \times 10^{-6} \text{ kg}$ have charges $Q_1 = -3.0 \text{ nC}$ and $Q_2 = -2.0 \text{ nC}$. They are released from rest when they were 40 cm from each other. Find the **distance between the particles** when the speed of each particle is 25 m/s? 5 points

Solution: Let r be the distance between the particles when their speeds are $3.0 \times 10^3 \text{ m/s}$. The work-energy principle gives

$$\begin{aligned} \text{KE}_f - \text{KE}_i + \text{PE}_f - \text{PE}_i &= 0 \\ \implies 2 \times \frac{1}{2}mv^2 - 0 + \frac{kQ_1Q_2}{r} - \frac{kQ_1Q_2}{0.4} &= 0 \\ \implies \frac{kQ_1Q_2}{r} = \frac{kQ_1Q_2}{0.4} - mv^2 &= 0.132 \\ \implies r &= 0.41 \text{ m} \end{aligned}$$

5. Two very large parallel flat sheets, A and B , are held at electric potentials $V_A = +20$ V and $V_B = -10$ V respectively as shown in the figure. The separation between the plates is $x = 8$ cm. The points i and f shown in the figure are also at a distance of x from each other and the angle $\theta = 40^\circ$. Find the potential difference ($V_i - V_f$). 3 points



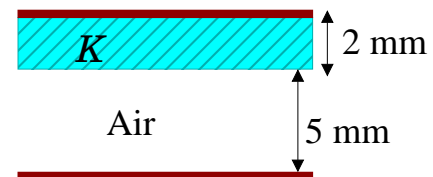
Solution: The electric field is **uniform**, so

$$V_A - V_B = Ex \implies E = \frac{V_A - V_B}{x} = 375 \text{ N/C}, \quad \text{and so, to the right}$$

The

$$V_i - V_f = Ex \cos 40^\circ = 23 \text{ V}$$

6. A parallel-plate capacitor with plate-area $A = 2.0 \times 10^{-4} \text{ m}^2$ is partially filled with a dielectric of dielectric constant $K = 3$ and thickness 2 mm. The thickness of empty space (air) between the plates is 5 mm (see the figure). Find the capacitance of this capacitor. 4 points



Solution: This capacitor can be considered as two capacitors C_1 (with the dielectric) and C_2 (with air) in **series**. Now

$$C_1 = K\epsilon_0 \frac{2.0 \times 10^{-4}}{0.002} = 2.66 \times 10^{-12} \text{ F}$$

$$C_2 = \epsilon_0 \frac{2.0 \times 10^{-4}}{0.005} = 3.54 \times 10^{-13} \text{ F}$$

The capacitance of the given capacitor is

$$C = \frac{C_1 C_2}{C_1 + C_2} = 3.13 \times 10^{-13} \text{ F}$$

7. A parallel-plate capacitor filled with a dielectric of dielectric constant $K = 4.3$ has a thickness $d = 4.0$ mm. The energy in the capacitor is $PE = 4.8 \times 10^{-6}$ J when it is connected to a battery. While it is still **connected to the battery**, the **dielectric is pulled out** and its **thickness is decreased** to $d' = 3.0$ mm. Find the energy now stored in this capacitor. 4 points

Solution: Let the old capacitor be C and the new capacitor be C' . Then

$$\frac{C'}{C} = \left(\frac{1.0}{K}\right) \left(\frac{A}{A}\right) \left(\frac{4.0 \times 10^{-3}}{3.0 \times 10^{-3}}\right) \implies \frac{C'}{C} = 0.31$$

In this case, the voltage V across the capacitor remains unchanged. So

$$\left. \begin{array}{l} PE = \frac{CV^2}{2} \\ PE' = \frac{C'V^2}{2} \end{array} \right\} \implies \frac{PE'}{PE} = \frac{C'}{C} = 0.31$$

$$\implies PE' = (0.31) \times PE = 1.5 \times 10^{-6} \text{ J}$$

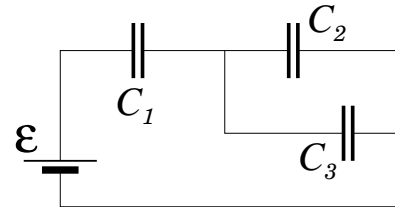
8. Three capacitors are connected to an unknown source of emf \mathcal{E} as shown in the circuit. The capacitance $C_1 = 6$ nF, $C_2 = 9$ nF, but C_3 is unknown. The plate-charges on C_1 and C_3 are respectively $Q_1 = 54$ nC and $Q_3 = 36$ nC.

(a) Find C_3 .

3 points

(b) Find \mathcal{E} .

2 points



Solution: We have

$$V_1 = \frac{Q_1}{C_1} = 9 \text{ V}$$

Then

$$\left. \begin{array}{l} C_2 \text{ and } C_3 \text{ are parallel} \\ C_1 \text{ and } C_{23} \text{ are in series} \end{array} \right\} \implies \left\{ \begin{array}{l} Q_1 = Q_{23} \implies Q_1 = Q_2 + Q_3 \\ Q_2 = Q_1 - Q_3 = 18 \text{ nC} \end{array} \right.$$

$$\implies V_2 = \frac{Q_2}{C_2} = 2 \text{ V}$$

$$\implies V_3 = V_2 = 2 \text{ V}$$

$$\implies C_3 = \frac{Q_3}{V_3} = 18 \text{ nF}$$

$$\implies \mathcal{E} = V_1 + V_{23} = V_1 + V_2 = 11 \text{ V}$$