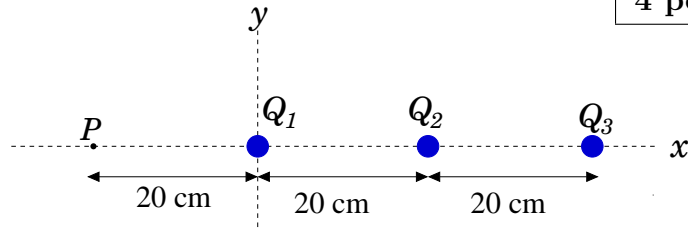


1. Three point charges are on the x -axis as shown. The net electric field $\vec{E} = 0$ at the point P . If $Q_1 = +2 \text{ nC}$ and $Q_2 = -6 \text{ nC}$, find Q_3 .

4 points



Solution: There are no y -components. We see that, $E_{1x} < 0$ and $E_{2x} > 0$. Then

$$E_x = E_{1x} + E_{2x} + E_{3x}$$

$$\Rightarrow 0 = -\frac{k|Q_1|}{0.2^2} + \frac{k|Q_2|}{0.4^2} + E_{3x} \Rightarrow 0 = -450.0 + 337.5 + E_{3x}$$

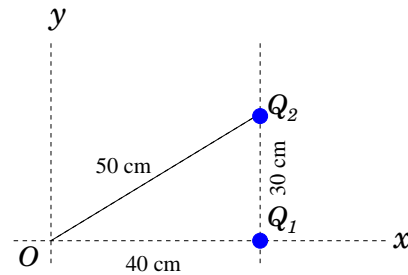
$$\Rightarrow E_{3x} = +112.5 \text{ N/C} \Rightarrow Q_3 < 0$$

Now $\frac{k|Q_3|}{0.6^2} = 112.5 \Rightarrow |Q_3| = 4.5 \times 10^{-9} \text{ C} \Rightarrow \boxed{Q_3 = -4.5 \times 10^{-9} \text{ C}}$

2. Two point charges, $Q_1 = -4.0 \text{ nC}$ and $Q_2 = +9.0 \text{ nC}$, are in the xy -plane as shown. Find the x -component and the y -component of the net electric field \vec{E} at O .

5 points

Solution: Since $Q_1 < 0$ and $Q_2 > 0$, the directions of \vec{E}_1 and \vec{E}_2 are as shown. The magnitudes and components of \vec{E}_1 and \vec{E}_2 are



$$E_1 = \frac{k|Q_1|}{0.4^2} = 225.0 \text{ N/C} \Rightarrow \begin{cases} E_{1x} = +225.0 \text{ N/C} \\ E_{1y} = 0 \text{ N/C} \end{cases}$$

$$E_2 = \frac{k|Q_2|}{0.5^2} = 324.0 \text{ N/C}; \Rightarrow \begin{cases} E_{2x} = -E_2 \times \frac{0.4}{0.5} = -259.2 \text{ N/C} \\ E_{2y} = -E_2 \times \frac{0.3}{0.5} = -194.4 \text{ N/C} \end{cases}$$

Then

$$E_x = E_{1x} + E_{2x} = -34.2 \text{ N/C}$$

$$E_y = E_{1y} + E_{2y} = -194.4 \text{ N/C}$$

3. Two small metallic balls with equal positive charge q and equal mass $m = 3.0 \times 10^{-6}$ kg each hang from the ceiling by two insulating strings of equal length. When the balls are in equilibrium, the strings make an angle $\theta = 20^\circ$ with the vertical and the distance between them is $d = 4.0$ cm. Find the charge q (Take $g = 9.8$ m/s²).

4 points

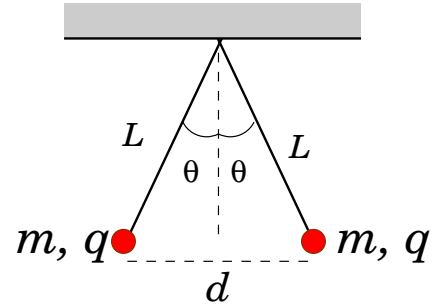
Solution: The free-body diagrams for the balls are shown. For any one ball, we find

$$\left. \begin{aligned} F_T \cos \theta &= mg \\ F_T \sin \theta &= F \end{aligned} \right\} \Rightarrow \tan \theta = \frac{F}{mg}$$

$$\Rightarrow F = mg \tan \theta = 1.07 \times 10^{-5} \text{ N}$$

Then

$$F = \frac{kq^2}{d^2} \Rightarrow q = \sqrt{\frac{Fd^2}{k}} = 1.38 \times 10^{-9} \text{ C}$$



4. A uniform ring of radius $a = 8$ cm and charge $Q = -4.0 \mu\text{C}$ is fixed in the yz -plane with its centre at the origin O . A point charge q is on the x -axis at $x = 12$ cm. The net electric potential $V = 0$ at O . Find the net electric field \vec{E} at O .

3 points

Solution: The net electric potential at O is

$$\frac{kQ}{a} + \frac{kq}{0.12} = 0$$

$$\Rightarrow q = -\frac{Q \times 0.12}{a} = +6.0 \times 10^{-6} \text{ C}$$

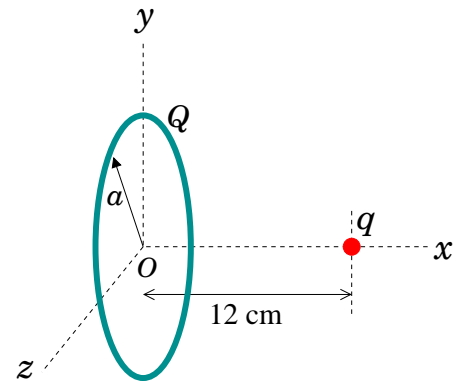
The electric field at O due to the ring is $E_{Ring} = 0$.

The electric field at O due to q is

$$E_q = \frac{k|q|}{(0.12)^2} = 3.75 \times 10^6 \text{ N/C, to the left}$$

So, the net electric field at O is

$$E = E_{Ring} + E_q = 3.75 \times 10^6 \text{ N/C, to the left}$$



5. Two charge particles of identical mass $m = 5.0 \times 10^{-14}$ kg are travelling directly towards each other. When the distance between them is 1.5 m, each particle is moving with speed $v = 3.0 \times 10^3$ m/s. If $q_1 = -2.0$ nC and the **distance of closest approach** between the particles is 20 cm, find q_2 .

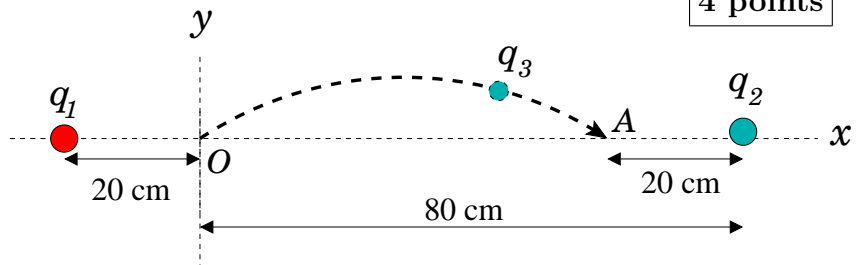
4 points

Solution: The speeds of the particles are zero at the distance of closest approach. The work-energy principle is

$$\begin{aligned} \text{KE}_i + \text{PE}_i &= \text{KE}_f + \text{PE}_f \\ \Rightarrow 2 \times \left(\frac{1}{2} m v^2 \right) + \frac{k q_1 q_2}{1.5} &= \frac{k q_1 q_2}{0.2} \\ \Rightarrow m v^2 &= q_2 \left(\frac{k q_1}{0.2} - \frac{k q_1}{1.5} \right) \\ \Rightarrow 4.5 \times 10^{-7} &= q_2 \times (-78.0) \\ \Rightarrow q_2 &= -5.8 \times 10^{-9} \text{ C} \end{aligned}$$

6. Two point charges $q_1 = -4.0 \mu\text{C}$ and $q_2 = +6.0 \mu\text{C}$ are fixed on the x -axis as shown. A third charged particle $q_3 = -2.0 \mu\text{C}$ is picked **from rest** at the origin O by an external agent and moved to point A . The kinetic energy of q_3 at A is $\text{KE}_A = 0.84$ J. How much work is done by the external agent?

4 points



Solution: The work-energy principle is

$$\begin{aligned} W_{ext} + \text{PE}_O + \text{KE}_O &= \text{PE}_A + \text{KE}_A \\ \Rightarrow W_{ext} + \left(\frac{k q_1 q_3}{0.2} + \frac{k q_2 q_3}{0.8} \right) + 0.0 &= \left(\frac{k q_1 q_3}{0.8} + \frac{k q_2 q_3}{0.2} \right) + \text{KE}_A \\ \Rightarrow W_{ext} + (0.36 - 0.135) + 0.0 &= (0.09 - 0.54) + 0.84 \\ \Rightarrow W_{ext} &= +0.165 \text{ J} \end{aligned}$$

7. A charged and isolated (**disconnected from the battery**) air-filled parallel-plate capacitor with thickness $d = 2.0$ mm has $PE = 4.8 \times 10^{-4}$ J of energy. If its thickness is increased to $d' = 5.0$ mm and a dielectric slab with $K = 3.4$ is inserted to fill the gap between the plates, find the energy now stored in this capacitor.

4 points

Solution: The new and old capacitances are related by

$$\frac{C'}{C} = \left(\frac{K}{1.0}\right) \left(\frac{A}{A}\right) \left(\frac{2.0 \times 10^{-3}}{5.0 \times 10^{-3}}\right) \Rightarrow \frac{C'}{C} = 1.36$$

In this case, the charge Q in the capacitor remains unchanged. So

$$\left. \begin{array}{l} PE = \frac{Q^2}{2C} \\ PE' = \frac{Q^2}{2C'} \end{array} \right\} \Rightarrow \frac{PE}{PE'} = \frac{C'}{C} = 1.36$$

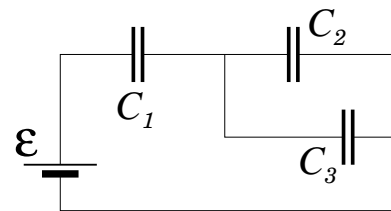
$$\Rightarrow PE' = \frac{PE}{1.36} = 3.5 \times 10^{-4} \text{ J}$$

8. Three capacitors are connected to a source of emf $\varepsilon = 15.0$ V as shown in the circuit. The capacitance C_1 is unknown, but $C_2 = 12.0$ nF and $C_3 = 20.0$ nF. If the plate-charge on C_3 is $Q_3 = 60$ nC, find C_1 .

5 points

Solution: We have

$$V_3 = \frac{Q_3}{C_3} = 3.0 \text{ V}$$



Then

$$C_2 \text{ and } C_3 \text{ are parallel} \Rightarrow C_{23} = C_2 + C_3 = 32.0 \text{ nF}$$

$$V_{23} = V_3 = 3.0 \text{ V}$$

$$Q_{23} = C_{23}V_{23} = 96.0 \text{ nC}$$

$$C_1 \text{ and } C_{23} \text{ are in series} \Rightarrow Q_1 = Q_{23} = 96.0 \text{ nC}$$

$$V_1 = \varepsilon - V_{23} = 12.0 \text{ V}$$

$$\Rightarrow C_1 = \frac{Q_1}{V_1} = 8.0 \text{ nF}$$