Kuwait University **Department of Physics**

جامعة الكويت
KUWAIT UNIVERSITY

General Physics II for Biological Sciences (Phy 122)

Summer Semester 2023-2024

Final Examination

July 30, 2024
Time: 11:00 AM to 1:00 PM

Instructor: Dr. S.S.A. Razee

Solution

Fundamental Constants

$k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$	Coulomb's constant
$\varepsilon_0 = 8.85 \times 10^{-12} \ \mathrm{C^2/(N{\cdot}m^2)}$	Permitivity of free space
$\mu_0 = 4\pi \times 10^{-7} \ \mathrm{T \cdot m/A}$	Permeability of free space
$e = 1.6 \times 10^{-19} \ \mathrm{C}$	Elementary unit of charge
$m_e = 9.11 \times 10^{-31} \ \mathrm{kg} = 0.000549 \ \mathrm{u}$	Mass of an electron
$m_p = 1.67 \times 10^{-27} \ \mathrm{kg} = 1.007276 \ \mathrm{u}$	Mass of a proton
$m_n = 1.67 \times 10^{-27} \ \mathrm{kg} = 1.008665 \ \mathrm{u}$	Mass of a neutron
$1 \ \mathrm{u} = 1.6605 \times 10^{-27} \ \mathrm{kg} = 931.5 \ \mathrm{MeV/c^2}$	Atomic mass unit
$1 \ \mathrm{eV} = 1.6 \times 10^{-19} \ \mathrm{J}$	Conversion from eV to J
$N_A = 6.022 \times 10^{23} \ \mathrm{/mol}$	Avogadro's number

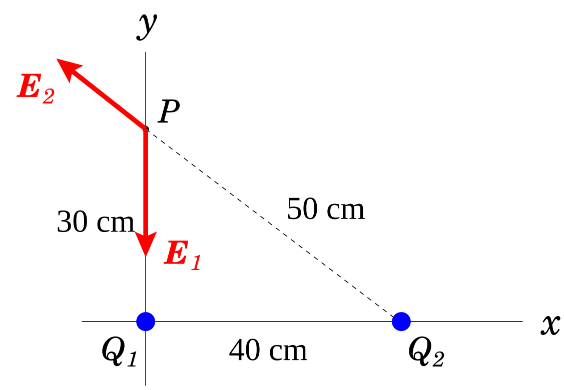
Prefixes of units

$\mathrm{m} = 10^{-3}$ $\mu = 10^{-6}$ $\mathrm{n} = 10^{-9}$ $\mathrm{p} = 10^{-12}$ $\mathrm{k} = 10^{3}$ $\mathrm{M} = 10^{6}$

Instructions to the Students:

- All communication devices must be switched off and placed in your bag. Anyone found using a communication device will be disqualified.
- Programmable calculators, which can store equations, are not allowed.

1. In the figure, Q_1 and Q_2 are point charges and the point P is on the $y-$axis. It is given that the net electric field at P has **no** $y-$**component** (E_y=0). If $Q_1 = -2.7$ nC, find the $\boldsymbol{x}-$**component** (E_x) of the **net electric field** at P. **4 points**



Solution: The electric field $\vec{\boldsymbol{E}}_1$ is towards Q_1 (since $Q_1 < 0$) as shown. Since the net y-component is zero, the only way we can draw $\vec{\boldsymbol{E}}_2$ is away from Q_2 as shown. This makes $\boldsymbol{Q_2 > 0}$.

Now,

$$E_{2y} - E_1 = 0$$

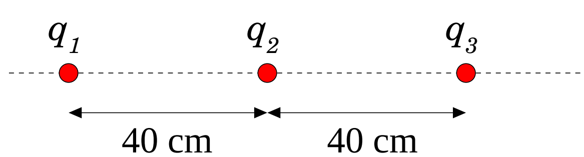
$$\Longrightarrow \frac{k|Q_2|}{(0.5)^2} \times \frac{0.3}{0.5} - \frac{k|Q_1|}{(0.3)^2} = 0 \quad \Longrightarrow \quad |Q_2| = \frac{|Q_1| \times (0.5)^3}{(0.3)^3} = 1.25 \times 10^{-8} \text{ C}$$

$$\Longrightarrow \boxed{Q_2 = +1.25 \times 10^{-8} \text{ C}}$$

Then

$$E_x = E_{2x} = -\frac{k|Q_2|}{(0.5)^2} \times \frac{0.4}{0.5} \quad \Longrightarrow \quad \boxed{E_x = -360 \text{ N/C}}$$

2. Three point particles of identical charge $q_1 = q_2 = q_3 = -2.0$ nC and identical mass $m = 3.0 \times 10^{-12}$ kg are initially on a straight line as shown. If they are released from rest at their positions, what will be the speed of the charge q_3 when it reaches infinity? **4 points**



Solution: Note the following:

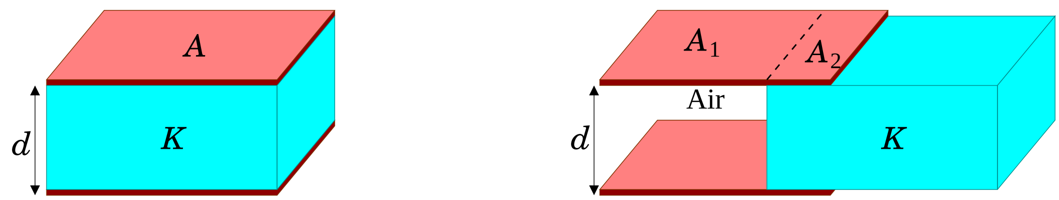
- $F_1 = F_3$, in opposite directions, and $F_2 = 0$
- So q_1 and q_3 are accelerates equally in opposite directions, q_3 remains at its place.
- So $v_1 = v_3 = v$, $v_2 = 0$.
- When q_3 is at infinity, q_1 is also at infinity. Then the electric potential energy is zero.

Then the work energy principle implies,

$$0 + \frac{kq^2}{0.4} + \frac{kq^2}{0.4} + \frac{kq^2}{0.8} = 2 \times \frac{1}{2}mv^2 + 0$$

$$\Longrightarrow 2.25 \times 10^{-9} = mv^2 \quad \Longrightarrow \quad \boxed{v = 274 \text{ m/s}}$$

3. A dielectric-filled (with $K = 3$) parallel-plate capacitor of thickness $d = 3$ mm and plate area $A = 6.0$ cm^2 (shown on the left) is charged to a voltage of $V = 15$ V and the **battery is disconnected**. Then the dielectric is partially pulled out (as shown on the right), such that the empty space between the plates has an area $A_1 = 4.0$ cm^2. Find the voltage across the capacitor **now**. **5 points**



Solution: The capacitance of the **original** capacitor is

$$C = K\varepsilon_0 \frac{A}{d} = 3 \times (8.85 \times 10^{-12}) \times \frac{6 \times 10^{-4}}{3.0 \times 10^{-3}} = 5.31 \times 10^{-12} \text{ F}$$

The new capacitor on the right can be considered as two capacitors C_1 and C_2 in parallel, with

$$C_1 = \varepsilon_0 \frac{A_1}{d} = (8.85 \times 10^{-12}) \times \frac{4 \times 10^{-4}}{3.0 \times 10^{-3}} = 1.18 \times 10^{-12} \text{ F}$$

$$C_2 = K\varepsilon_0 \frac{A_2}{d} = 3 \times (8.85 \times 10^{-12}) \times \frac{2 \times 10^{-4}}{3.0 \times 10^{-3}} = 1.77 \times 10^{-12} \text{ F}$$

The new capacitance C' is

$$C' = C_1 + C_2 = 2.95 \times 10^{-12} \text{ F}$$

The **battery is disconnected**, so the **plate-charge Q remains the same**.

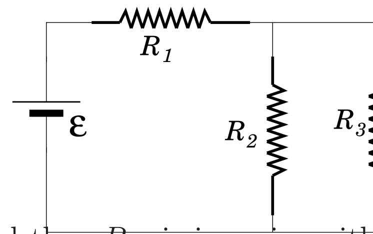
$$Q = CV = 7.965 \times 10^{-11} \text{ C}$$

$$Q = C'V' \implies V' = \frac{Q}{C'} = 27 \text{ V}$$

4. A circuit is shown, $\varepsilon = 90$ V. The power dissipated by R_2 and R_3 are $P_2 = 50$ W and $P_3 = 100$ W, and the current in R_1 is $I_1 = 2.5$ A.

   (a) Find R_1. **3 points**

   (b) Find R_2. **2 points**



Solution: We observe that R_2 and R_3 are parallel and then R_{23} is in series with I_1. So the total current $\boldsymbol{I_{123} = I_1}$.

The total power supplied by the battery is

$$I_{123}\varepsilon = P_1 + P_2 + P_3 \implies P_1 = I_1\varepsilon - P_2 - P_3 \implies \boldsymbol{P_1 = 75} \text{ W}$$

Then

$$P_1 = I_1^2 R_1 \implies \boldsymbol{R_1 = 12\ \Omega}$$

We have

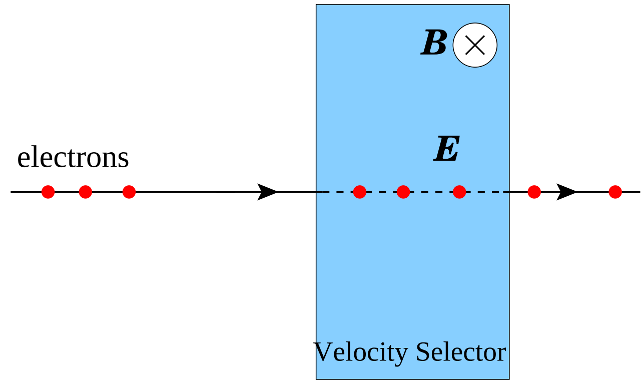
$$R_{23} \text{ and } R_1 \text{ are in series} \implies \varepsilon = V_1 + V_{23}$$

$$\implies V_{23} = \varepsilon - V_1 = \varepsilon - I_1 R_1 = 60 \text{ V}$$

$$R_2 \text{ and } R_3 \text{ are parallel} \implies V_2 = V_{23} = 60 \text{ V}$$

$$P_2 = \frac{V_2^2}{R_2} \implies R_2 = \frac{V_2^2}{P_2} \implies \boldsymbol{R_2 = 72\ \Omega}$$

5. A beam of electrons goes **undeflected** when it passes through a velocity selector where the magnetic field $\vec{B}$ is **into-the-plane** as shown.



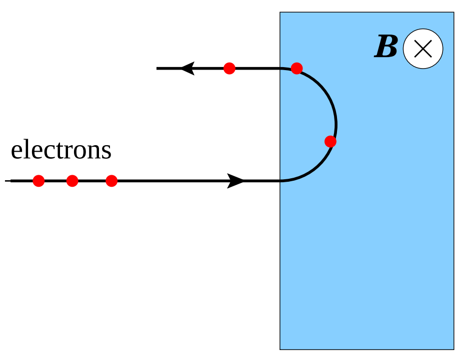
(a) The direction of $\vec{E}$ is (tick the correct answer) **1 point**

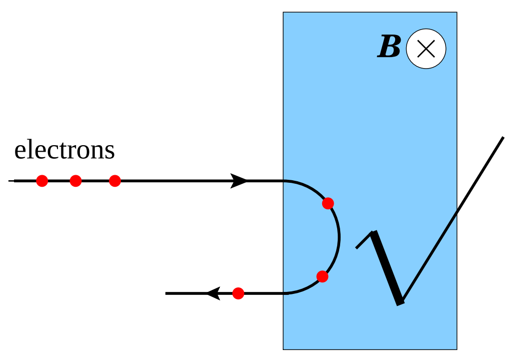
upward ($\uparrow$) | **downward** ($\downarrow$) ✓ | **to the right** ($\longrightarrow$) | **to the left** ($\longleftarrow$)

(b) The magnitudes of $\vec{E}$ and $\vec{B}$ are $E = 3.4 \times 10^4$ N/C and $B = 2.5 \times 10^{-3}$ T. Find the speed v of the electrons. **1 point**

$$v = \frac{E}{B} = 1.36 \times 10^7 \text{ m/s}$$

(c) If the electric field $\vec{E}$ is **switched off**, the electrons will move in **semi-circular paths**. The semicircular path is (tick the correct figure) **1 point**

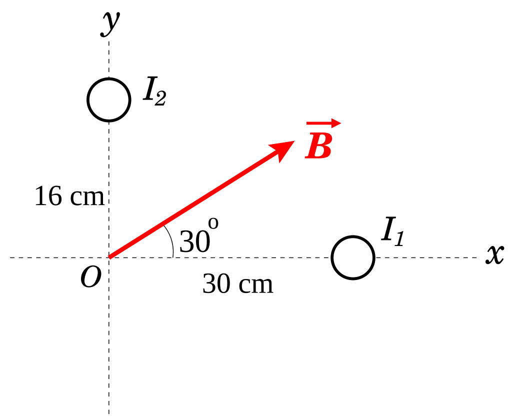




(d) Find the radius R of the semicircular paths. **2 points**

$$R = \frac{mv}{B|q|} = 0.031 \text{ m}$$

6. Two long straight wires perpendicular to the $xy-$plane are shown. The wire with current I_1 is passing through the point $x = 30$ cm on the x-axis and the wire with current I_2 is passing through the point $y = 16$ cm on the y-axis (see the figure). The net magnetic field $\vec{B}$ at the origin producs by these currents has a magnitude $B = 8.2 \times 10^{-6}$ T and it makes an angle 30^o with the positive x-axis as shown.



(a) The current I_1 is (tick the correct answer) **1 point**

out-of-the-plane ($\odot$) | **into-the-plane** ($\otimes$) ✓

(b) The current I_2 is (tick the correct answer) **1 point**

out-of-the-plane ($\odot$) ✓ | **into-the-plane** ($\otimes$)

(c) Find the $x-$component (B_x) of the magnetic field. **1 point**

$$B_x = +B\ \cos 30^o = 7.1 \times 10^{-6}\ \text{T}$$

(d) Find the $y-$component (B_y) of the magnetic field. **1 point**

$$B_y = +B\ \sin 30^o = 4.1 \times 10^{-6}\ \text{T}$$

(e) Find the value of the current I_1. **1 point**

$\boldsymbol{B_y}$ is due to $\boldsymbol{I_1}$. So

$$\frac{\mu_0 I_1}{2\pi(0.30)} = 4.1 \times 10^{-6} \implies I_1 = 6.15\ \text{A}$$

(f) Find the value of the current I_2. **1 point**

$\boldsymbol{B_x}$ is due to $\boldsymbol{I_2}$. So

$$\frac{\mu_0 I_2}{2\pi(0.16)} = 7.1 \times 10^{-6} \implies I_2 = 5.68\ \text{A}$$

7. A concave mirror produces a real image 2 times the size of the object. If the focal length of the mirror is 20 cm, find the object distance d_o. **4 points**

Solution: It is 2× real image, so

$$m = -2 \implies \frac{-d_i}{d_o} = -2 \implies d_i = 2d_o$$

Then, the mirror equation is

$$\frac{1}{d_o} + \frac{1}{2d_o} = \frac{1}{f} \implies \frac{1.5}{d_o} = \frac{1}{f}$$

$$\implies d_o = 1.5 \times f = 0.3 \text{ m}$$

8. A near-sighted person has his **near-point** at 17 cm and **far-point** at 66 cm. He wants to wear contact lenses.

   (a) To correct his vision, what power of lense is advisable? **2 points**

   (b) Can he wear the same glasses while reading? **2 points**

Solution: He needs correcting lenses for his **far vision only**, because the minimum distance of normal vision (25 cm) is within his range of vision. The distant objects ($d_o \to \infty$) need to have their **virtual images** at 57 cm. So

$$0 + \frac{1}{-0.66} = \frac{1}{f} = P \implies P = -1.5 \text{ D}$$

While reading, he is supposed to hold the book at 25 cm (distance of normal vision). Then

$$\frac{1}{0.25} + \frac{1}{d_i} = -1.5 \implies d_i = -0.18 \text{ m}$$

So it will produce a virtual image at 18 cm which is well within his range of vision. So **yes**, he can use the same glasses for reading as well. However, he can read without the glasses as well.

9. Consider the radioactive isotope ${}^{40}_{19}\mathrm{K}$.

   (a) Determine whether it is possible for ${}^{40}_{19}\mathrm{K}$ to emit an α particle. **2 points**

   For α decay, the equation is

$${}^{40}_{19}\mathrm{K} \longrightarrow {}^{36}_{17}\mathrm{K} + {}^{4}_{2}\mathrm{He}$$

   So the Q-value is

$$Q = [39.953567 - 35.958045 - 4.001505] \times 931.5 \text{ MeV} = -5.57 \text{ MeV}$$

   Since $Q < 0$, emission of α particles is **not possible**.

   (b) Determine whether it is possible for ${}^{40}_{19}\mathrm{K}$ to emit a β^+ particle. **2 points**

   For β^+ decay, the equation is

$${}^{40}_{19}\mathrm{K} \longrightarrow {}^{40}_{18}\mathrm{Ar} + e^+$$

   So the Q-value is

$$Q = [39.953567 - 39.952501 - 0.000549] \times 931.5 \text{ MeV} = +0.48 \text{ MeV}$$

   Since $Q > 0$, emission of β^+ particles is **possible**.

   (c) Determine whether it is possible for ${}^{40}_{19}\mathrm{K}$ to emit a β^- particle. **2 points**

For β^- decay, the equation is

$${}^{40}_{19}\mathrm{K} \longrightarrow {}^{40}_{20}\mathrm{Ca} + e^-$$

So the Q-value is

$$Q = [39.953567 - 39.951611 - 0.000549] \times 931.5 \text{ MeV} = +1.31 \text{ MeV}$$

Since $Q > 0$, emission of β^- particles is **possible**.

The nuclear mass of some isotopes are given here:

${}^{40}_{19}\mathrm{K}$: 39.953567 u	${}^{40}_{18}\mathrm{Ar}$: 39.952501 u	${}^{40}_{20}\mathrm{Ca}$: 39.951611 u
${}^{36}_{17}\mathrm{Cl}$: 35.958045 u	${}^{39}_{18}\mathrm{Ar}$: 38.943836 u	${}^{39}_{20}\mathrm{Ca}$: 38.942946 u
${}^{4}_{2}\mathrm{He}$: 4.001505 u	$m_e = 0.000549$ u	

10. The **activity** of a sample containing $^{222}_{86}$Ra decreases from (1.6×10^6) decays per second to (2.6×10^5) decays per second in 10 complete days.

(a) Find the **half-life** of $^{222}_{86}$Ra. **2 points**

The time

$$t = 10 \text{ days } = 10 \times 24 \times 3600 = 8.64 \times 10^5 \text{ s}$$

The activity equation is

$$R = R_0 \, e^{-\lambda t} \implies e^{-\lambda t} = \frac{R}{R_0} = 0.1625$$

$$\implies \lambda t = -\ln(0.1625) \implies \lambda = \frac{-\ln(0.1625)}{t} = 2.1 \times 10^{-6} \text{ s}^{-1}$$

$$T_{\frac{1}{2}} = \frac{0.693}{\lambda} = 3.295 \times 10^5 \text{ s}$$

(b) Find the initial number of $^{222}_{86}$Ra atoms present in the sample. **1 point**

$$R_0 = \lambda N_0 \longrightarrow N_0 = \frac{R_0}{\lambda} = 7.62 \times 10^{11}$$

(c) Find the initial **mass** of $^{222}_{86}$Ra atoms in the sample. **2 points**

The initial mass is

$$\text{Mass } = \frac{N_o}{6.02 \times 10^{23}} \times 222 \text{ g} = 2.81 \times 10^{-10} \text{ g}$$