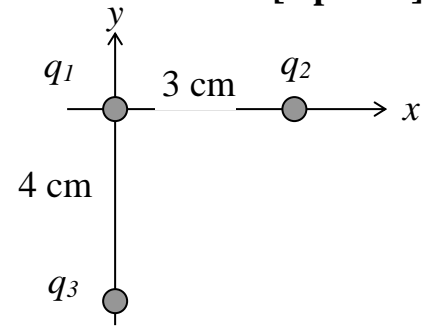


1. Three charges $q_1 = 10 \mu\text{C}$, $q_2 = 20 \mu\text{C}$ and $q_3 = -30 \mu\text{C}$ are located in the x - y plane as shown in the figure. What is the magnitude of the resultant force on the charge q_2 ?

[5 points]



$$F_{21} = k \frac{|q_2||q_1|}{r_{21}^2} = \frac{9 \times 10^9 \times 20 \times 10^{-6} \times 10 \times 10^{-6}}{(0.03)^2}$$

$$= 2000 \text{ N/C}$$

$$F_{23} = k \frac{|q_2||q_3|}{r_{23}^2} = \frac{9 \times 10^9 \times 20 \times 10^{-6} \times 30 \times 10^{-6}}{(0.03^2 + 0.04^2)}$$

$$= 2160 \text{ N/C}$$

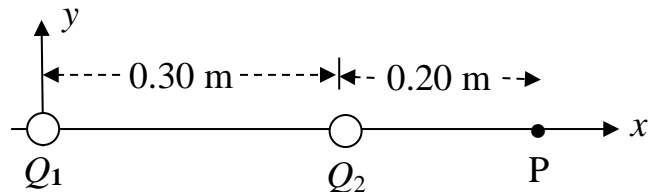
$$F_x = F_{21} - F_{23} \cos \theta = 2000 - 2160 \times \frac{0.03}{0.05} = 704 \text{ N/C}$$

$$F_y = -F_{23} \sin \theta = -2160 \times \frac{0.04}{0.05} = -1728 \text{ N/C}$$

$$F_{net} = \sqrt{F_x^2 + F_y^2} = 1866 \text{ N}$$

2. Two charges $Q_1 = 50 \text{ nC}$ and Q_2 are located as shown in the figure. What should be the magnitude and sign of charge Q_2 so that the net electric field at the point P is $E = 675 \text{ N/C}$ directed along $+x$ -axis?

[3 points]



$$E_1 = k \frac{|Q_1|}{(0.30+0.20)^2} = 1800 \text{ N along } +x \text{ axis}$$

$$E_2 = k \frac{|Q_2|}{(0.20)^2}$$

$$E = E_1 + E_2$$

$$675 = 1800 + E_2$$

$$E_2 = 675 - 1800 = -1125 \text{ N/C (must be along -x-axis)}$$

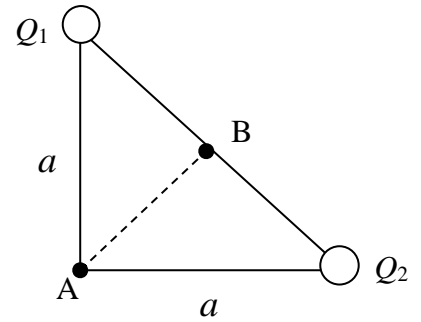
$$k \frac{|Q_2|}{(0.20)^2} = -1125 \rightarrow$$

$$Q_2 = -5 \times 10^{-9} \text{ C} = 5.0 \text{ nC}$$

3. Two charges $Q_1 = 20 \mu\text{C}$ and $Q_2 = -15 \mu\text{C}$ are located at the corners of a right-angled triangle, with $a = 0.30 \text{ m}$, as shown in the figure.

(a) What is the potential difference $V_B - V_A$? [4 points]

(b) How much work must be done to move charge $Q = 5 \mu\text{C}$ from A to B? [1 point]



$$(a) r_{1B} = r_{2B} = \frac{a\sqrt{2}}{2} = 0.21 \text{ m}$$

$$V_A = V_{Q_1} + V_{Q_2} = k \frac{Q_1}{0.30} + k \frac{Q_2}{0.30} = 1.5 \times 10^5 \text{ V}$$

$$V_B = V_{Q_1} + V_{Q_2} = k \frac{Q_1}{0.21} + k \frac{Q_2}{0.21} = 2.1 \times 10^5 \text{ V}$$

$$V_B - V_A = 0.6 \times 10^5 \text{ V}$$

$$(b) W_{ext} = Q\Delta V = Q(V_B - V_A) = 0.30 \text{ J}$$

4. Two capacitors $C_1 = 8 \mu\text{F}$ and $C_2 = 12 \mu\text{F}$ are connected in series across a 10 V battery, as shown. The battery is disconnected, and the capacitors are then connected in parallel with positive plates connected to each other and negative plates connected together. What is the charge across C_1 now? [4 points]

$$C_s = \frac{C_1 C_2}{C_1 + C_2} = \frac{8 \mu\text{F} \cdot 12 \mu\text{F}}{8 \mu\text{F} + 12 \mu\text{F}} = 4.8 \mu\text{F}$$

Initial charge Q on C_1 and C_2 :

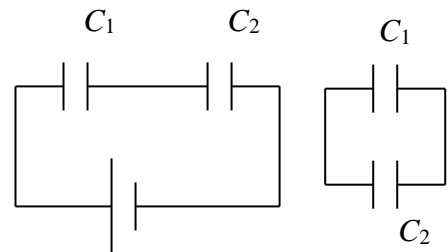
$$Q = C_{eq} V = 4.8 \mu\text{F} \times 10 \text{ V} = 48 \mu\text{C}$$

When connected in parallel:

Voltage across C_1 and C_2 :

$$V = \frac{Q_{total}}{C_{eq}} = \frac{48 \mu\text{C} + 48 \mu\text{C}}{(8 \mu\text{F} + 12 \mu\text{F})} = 4.8 \text{ V}$$

$$q_1 = C_1 V = 8 \mu\text{F} \times 4.8 \text{ V} = 38.4 \mu\text{C}$$



5. A resistor $R = 7.8 \Omega$ is connected to a battery with an internal resistance r and the emf $\varepsilon = 6 \text{ V}$. The energy dissipated in the resistor R in 5 minutes is 0.96 kJ .

(a) What is the rate of power dissipated in the resistor R ?

[1 point]

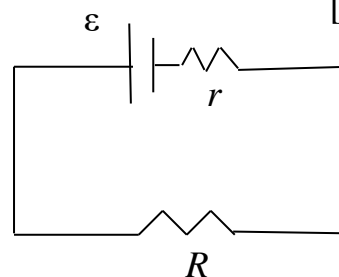
(b) What is the current in the circuit?

[1 point]

(c) What is the internal resistance r ?

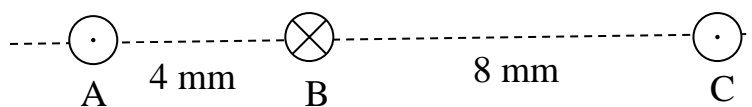
[1 point]

$$\begin{aligned} \text{(a)} \quad P &= \frac{E}{t} = \frac{0.96 \times 10^3 \text{ J}}{5 \times 60 \text{ s}} = 3.2 \text{ W} \\ \text{(b)} \quad P &= I^2 R \rightarrow I = \sqrt{\frac{P}{R}} = 0.64 \text{ A} \\ \text{(c)} \quad I &= \frac{\varepsilon}{R+r} \\ R+r &= \frac{\varepsilon}{I} = 9.38 \Omega \\ r &= 9.38 - R = 1.58 \Omega \end{aligned}$$



6. Three parallel wires are placed perpendicular to the page, each carrying a current of $I = 12 \text{ A}$, as shown. The current in wires A and C is out of the page and in wire B it is into the page. What is the magnitude and direction of the resultant force on 2 m length of the wire C?

[3 points]



$$\begin{aligned} F_{CA} &= \frac{\mu_0 I_C I_A L}{2\pi d} = 4.8 \times 10^{-3} \text{ N along } -x \text{ axis} \\ F_{CB} &= \frac{\mu_0 I_C I_B L}{2\pi d} = 7.2 \times 10^{-3} \text{ N along } +x \text{ axis} \\ F_{net} &= F_{CB} - F_{CA} = 2.4 \times 10^{-3} \text{ N along } +x \text{ axis} \end{aligned}$$

7. Two long parallel conductors carry currents $I_1 = 8.0 \text{ A}$ and $I_2 = 20.0 \text{ A}$ in the directions shown. What is the x -component of the net magnetic field at point P? [4 points]

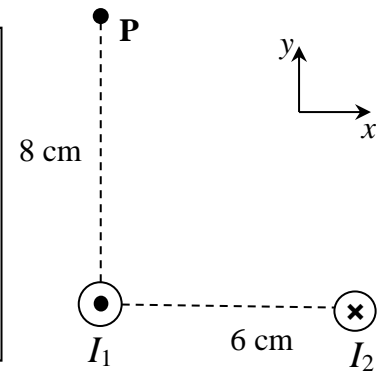
$$B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{4\pi \times 10^{-7} \times 8.0}{2\pi \times 0.08} = 2.0 \times 10^{-5} \text{ T along } -x \text{ axis}$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{4\pi \times 10^{-7} \times 20.0}{2\pi \times \sqrt{0.08^2 + 0.06^2}} = 4.0 \times 10^{-5} \text{ T}$$

at angle θ above x -axis

$$B_x = -B_1 + B_2 \cos \theta = -2.0 \times 10^{-5} \text{ T} + 4.0 \times 10^{-5} \text{ T} \times \frac{0.08}{0.10}$$

$$= 1.2 \times 10^{-5} \text{ T}$$



8. An object of height 5.0 cm is placed 10.0 cm from a lens with a power of +5.00 D.
- What is the focal length of the lens? [1 point]
 - What will be the location of the image? [1 points]
 - Will the image be inverted or upright? [1 point]
 - What will be the height of the image? [1 point]
 - Is the image real or imaginary? [1 point]

$$(a) f = \frac{1}{P} = \frac{1}{5.00 \text{ m}^{-1}} = 0.20 \text{ m} = 20 \text{ cm}$$

$$(b) \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{20 \text{ cm}} - \frac{1}{10 \text{ cm}} = -0.05 \text{ cm}^{-1}$$

$$d_i = -20.0 \text{ cm on the same side as object}$$

$$(c) m = -\frac{d_i}{d_o} = -\frac{-20.0 \text{ cm}}{10.0 \text{ cm}} = 2.0 \text{ the image is upright and magnified.}$$

$$(d) h_i = mh_o = 2.0 \times 5.0 \text{ cm} = 10.0 \text{ cm}$$

(e) The image is imaginary.

9. The binding energy of the last neutron in a nucleus of ^{15}N is 10.83 MeV.
Given $M(^{14}\text{N}) = 14.003074 \text{ u}$, $M(\text{n}) = 1.008665 \text{ u}$, and $M(^1\text{H}) = 1.007825 \text{ u}$.

(a) What is the mass of ^{15}N ?

[2 points]

(b) What is the binding energy per nucleon of ^{15}N ?

[2 points]

(a) Binding energy of the last neutron in ^{15}N :

$$^{15}\text{N} = ^{14}\text{N} + \text{n}$$

$$\text{BE}_n = (M(^{14}\text{N}) + M(\text{n}) - M(^{15}\text{N}))c^2$$

$$10.83 \text{ MeV} = (14.003074 \text{ u} + 1.008665 \text{ u} - M(^{15}\text{N})) \times 931.5 \text{ MeV}$$

$$M(^{15}\text{N}) = 15.000113 \text{ u}$$

(b) $^{15}\text{N} = 7(^1\text{H}) + 8\text{n}$

$$\text{BE} (^{15}\text{N}) = (7 \times M(^1\text{H}) + 8 \times M(\text{n}) - M(^{15}\text{N}))c^2$$

$$= (7 \times 1.007825 \text{ u} + 8 \times 1.008665 \text{ u} - 15.000113 \text{ u}) \times 931.5 \text{ MeV}$$

$$= 115.49 \text{ MeV}$$

$$\frac{\text{BE}}{A} = \frac{115.49 \text{ MeV}}{15} = 7.699 \text{ MeV/nucleon}$$

10. For the isotope ^{18}F (half-life = 110 min.) given to a patient in the Positron Emission Tomography (PET), the decay rate becomes 180 per second after 30 hours.

(a) What was the decay rate initially?

[2 points]

(b) How many ^{18}F nuclei were present in the beginning?

[1 point]

(c) How much amount of ^{18}F (in gram) was present initially?

[1 point]

$$(a) \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{110 \times 60 \text{ s}} = 1.05 \times 10^{-4} \text{ s}^{-1}$$

$$R = R_0 e^{-\lambda t}$$

$$R_0 = R e^{\lambda t} = 180 \text{ s}^{-1} \times e^{1.05 \times 10^{-4} \text{ s}^{-1} \times (30 \times 60 \times 60 \text{ s})}$$

$$= 1.51 \times 10^7 \text{ s}^{-1}$$

$$(b) R_0 = \lambda N_0$$

$$N_0 = \frac{R_0}{\lambda} = \frac{1.51 \times 10^7 \text{ s}^{-1}}{1.05 \times 10^{-4} \text{ s}^{-1}} = 1.44 \times 10^{11} \text{ nuclei}$$

$$(c) N = \frac{N_A}{\text{Mol (gram)}} \times m(\text{gram})$$

$$m(\text{gram}) = \frac{N \times \text{Mol (gram)}}{N_A}$$

$$= \frac{1.44 \times 10^{11} \times 15}{6.02 \times 10^{23}} = 3.59 \times 10^{-12} \text{ gram}$$