



**Second Midterm Examination  
Summer Semester 2023 – 2024**

**July 20, 2024**

**Time: 11:30 – 1:00 PM**

Name: ..... Student No: .....

Section No: ..... Serial No: .....

Instructors: Drs. Alfailakawi, Lajko, and Vagenas

**Fundamental constants**

$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2$	(Coulomb constant)
$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)$	(Permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m}/\text{A}$	(Permeability of free space)
$ e  = 1.60 \times 10^{-19} \text{ C}$	(Elementary unit of charge)
$N_A = 6.02 \times 10^{23}$	(Avogadro's number)
$g = 9.8 \text{ m/s}^2$	(Acceleration due to gravity)
$m_e = 9.11 \times 10^{-31} \text{ kg}$	(Electron mass)
$m_p = 1.67 \times 10^{-27} \text{ kg}$	(Proton mass)

**Prefixes of units**

$m = 10^{-3}$	$\mu = 10^{-6}$	$n = 10^{-9}$	$p = 10^{-12}$
$k = 10^3$	$M = 10^6$	$G = 10^9$	$T = 10^{12}$

**For use by Instructors only**

Problems	1	2	3	4	5	6	7	8	Questions	Total
Marks										

**Instructions to the Students:**

1. Mobile or other electronic devices are **strictly prohibited** during the exam.
2. Programmable calculators, which can store equations, are not allowed.
3. **Cheating incidents will be processed according to the university rules.**

**PART I: Solve the following problems. Show your solutions in detail.**

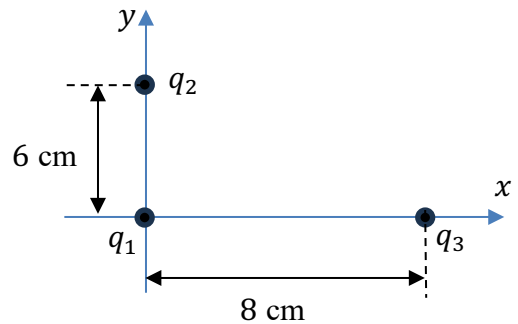
1. Three point charges  $q_1 = q_2 = 10 \text{ nC}$ , and  $q_3$  are placed on the  $x$  and  $y$ -axis as shown. The total potential energy of the system is  $20 \text{ }\mu\text{J}$ . Find the electric charge  $q_3$ . **[4 points]**

$$d = \sqrt{(0.06)^2 + (0.08)^2} \text{ m} \Rightarrow d = 0.10 \text{ m}$$

$$U_{total} = k \frac{q_1 q_2}{0.06} + k \frac{q_1 q_3}{0.08} + k \frac{q_2 q_3}{0.10}$$

$$20 \times 10^{-6} = 15 \times 10^{-6} J + 1125 q_3 + 900 q_3$$

$$q_3 = 2.47 \text{ nC}$$

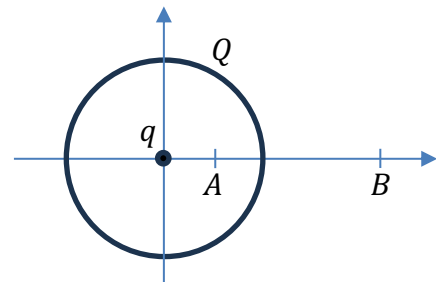


2. A conducting sphere of radius  $R = 0.4 \text{ m}$  is uniformly charged with electric charge  $Q = 30 \text{ nC}$ . A point charge  $q = 5 \text{ nC}$  is placed at the center of the sphere. Find the potential difference  $V_A - V_B$ .  
Given:  $x_A = 0.2 \text{ m}$  and  $x_B = 0.9 \text{ m}$ . **[3 points]**

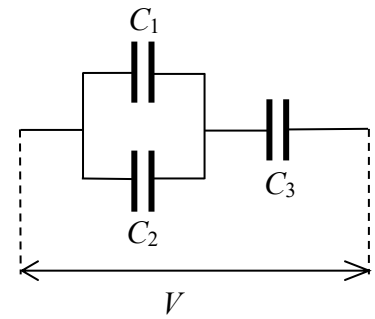
$$V_A = k \frac{Q}{R} + k \frac{q}{x_A} \Rightarrow V_A = 900 \text{ V}$$

$$V_B = k \frac{Q}{x_B} + k \frac{q}{x_B} \Rightarrow V_B = 350 \text{ V}$$

$$V_A - V_B = 550 \text{ V}$$



3. In the capacitor network shown,  $C_1 = C_2 = 5 \mu\text{F}$ ,  $C_3 = 30 \mu\text{F}$ , and the applied potential difference is  $V = 4.8 \text{ V}$ . Find the electric potential energy stored in capacitor  $C_1$ . **[5 points]**



**$C_1$  and  $C_2$  are in parallel:**

$$C_{12} = C_1 + C_2 \Rightarrow C_{12} = 10 \mu\text{F}$$

**$C_{12}$  and  $C_3$  are in series:**

$$C_{123} = \frac{C_{12}C_3}{C_{12}+C_3} \Rightarrow C_{123} = 7.5 \mu\text{F}$$

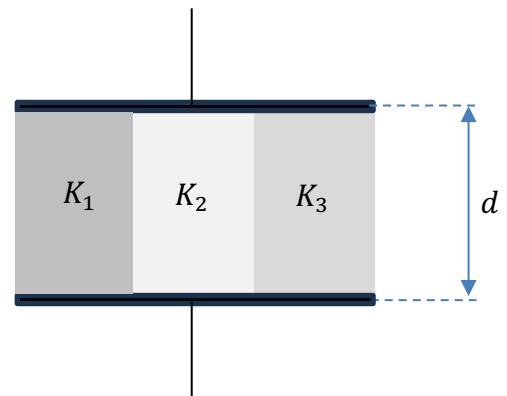
The total charge of  $C_{123}$  is  $Q_{123} = Q_{12} = Q_3$

$$C_{123} = \frac{Q_{12}}{V} \Rightarrow Q_{12} = C_{123}V \Rightarrow Q_{12} = 36 \mu\text{C}$$

$$C_{12} = \frac{Q_{12}}{V_{12}} \Rightarrow V_{12} = 3.6 \text{ V}$$

$$U_1 = \frac{1}{2}C_1V_{12}^2 \Rightarrow U_1 = 32.4 \mu\text{J}$$

4. An air-filled parallel-plate capacitor with a plate area  $A$  and separation  $d$ , has capacitance  $C_0$ . Then, the capacitor is partially filled with three dielectric slabs with constants  $K_1 = 4$ ,  $K_2 = 5$ ,  $K_3 = 6$ , as shown. Each dielectric slab fills  $1/3$  of the space between the capacitor plates. If the equivalent capacitance is  $200 \mu\text{F}$ , find the initial capacitance  $C_0$ . **[5 points]**



$$C_0 = \epsilon_0 \frac{A}{d}$$

$$C_1 = K_1 \epsilon_0 \frac{A/3}{d} \Rightarrow C_1 = \frac{K_1}{3} \epsilon_0 \frac{A}{d} \Rightarrow C_1 = \frac{4}{3} C_0$$

$$C_2 = K_2 \epsilon_0 \frac{A/3}{d} \Rightarrow C_2 = \frac{K_2}{3} \epsilon_0 \frac{A}{d} \Rightarrow C_2 = \frac{5}{3} C_0$$

$$C_3 = K_3 \epsilon_0 \frac{A/3}{d} \Rightarrow C_3 = \frac{K_3}{3} \epsilon_0 \frac{A}{d} \Rightarrow C_3 = \frac{6}{3} C_0$$

**$C_1, C_2,$  and  $C_3$  are in parallel:**

$$C_{eq} = C_1 + C_2 + C_3 \Rightarrow C_{eq} = \frac{4}{3}C_0 + \frac{5}{3}C_0 + \frac{6}{3}C_0 \Rightarrow C_{eq} = \frac{15}{3}C_0 \Rightarrow C_0 = 40 \mu\text{F}$$

5. A gold wire of length  $L = 3$  m has resistivity  $\rho = 2.44 \times 10^{-8} \Omega \cdot \text{m}$  and concentration of free electrons  $n = 5.90 \times 10^{28} \text{ m}^{-3}$ . When a potential difference  $V$  is applied, the electrons run the full length of the gold wire in time  $2 \times 10^4$  s. Find the potential difference  $V$ . **[4 points]**

$$v_d = \frac{L}{t} \Rightarrow v_d = \frac{3 \text{ m}}{2 \times 10^4 \text{ s}} \Rightarrow v_d = 1.5 \times 10^{-4} \text{ m/s}$$

$$J = n|e|v_d \Rightarrow J = 14.16 \times 10^5 \text{ A/m}^2$$

$$E = \rho J \Rightarrow E = 0.0346 \text{ N/C}$$

$$E = \frac{V}{L} \Rightarrow V = EL \Rightarrow V = 0.104 \text{ V}$$

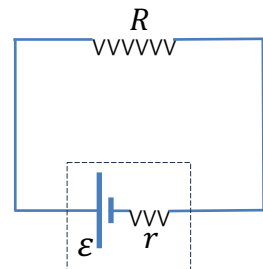
6. In the electric circuit below, the power dissipated on the resistor  $R = 3 \Omega$  is  $P_R = 15$  W. The internal resistance of the battery is  $r = 1 \Omega$ . Find the emf  $\mathcal{E}$  of the battery. **[3 points]**

$$P_R = I^2 R \Rightarrow I = \sqrt{\frac{P_R}{R}} \Rightarrow I = 2.24 \text{ A}$$

$$P_R = \frac{V_R^2}{R} \Rightarrow V_R = \sqrt{P_R R} \Rightarrow V_R = 6.71 \text{ V}$$

Since  $V_R = V_{\text{terminal}}$

$$V_{\text{terminal}} = \mathcal{E} - Ir \Rightarrow \mathcal{E} = V_R + Ir \Rightarrow \mathcal{E} = 8.95 \text{ V}$$



7. Find the electric currents  $I_1$ ,  $I_2$ , and  $I_3$  in the electric circuit below.

[5 points]

**Junction rule:**  $I_3 = I_1 + I_2$

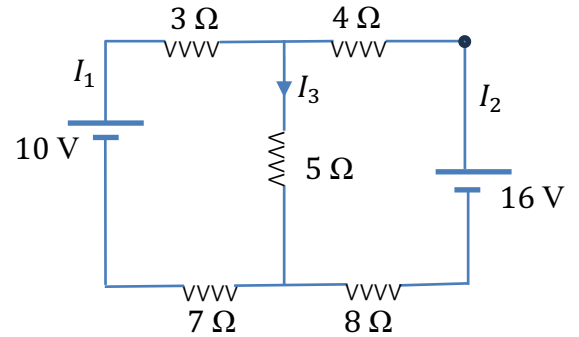
**Loop rule:**

- **Left loop**

$$10 - 3I_1 - 5I_3 - 7I_1 = 0 \Rightarrow 10 - 15I_1 - 5I_2 = 0$$

- **Right loop**

$$16 - 4I_2 - 5I_3 - 8I_2 = 0 \Rightarrow 16 - 5I_1 - 17I_2 = 0$$



Multiply 2<sup>nd</sup> loop rule equation with (-3):

$$-48 + 15I_1 + 51I_2 = 0$$

and then add it to the 1<sup>st</sup> loop rule. We get:

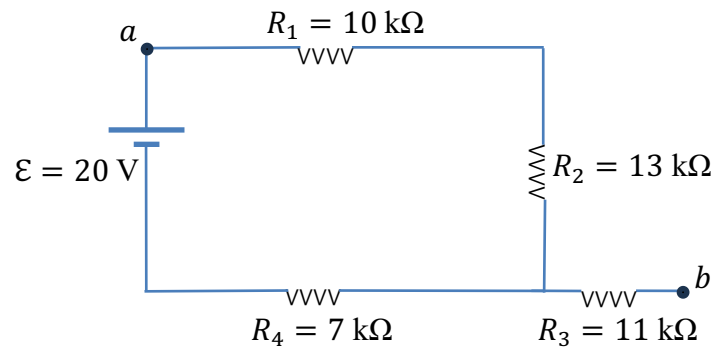
$$-38 + 46I_2 = 0 \Rightarrow I_2 = 0.826 \text{ A}$$

$$10 - 15I_1 - 5 \times 0.826 = 0 \Rightarrow I_1 = 0.391 \text{ A}$$

$$I_3 = (0.391 + 0.826) \text{ A} \Rightarrow I_3 = 1.217 \text{ A}$$

8. Find the potential difference between points  $a$  and  $b$  in the electric circuit below.

[3 Points]



**Loop rule:**

$$\mathcal{E} - IR_1 - IR_2 - IR_4 = 0 \Rightarrow I = \frac{\mathcal{E}}{R_1 + R_2 + R_4} \Rightarrow I = 0.67 \times 10^{-3} \text{ A}$$

$$V_a - IR_1 - IR_2 = V_b \Rightarrow V_a - V_b = I(R_1 + R_2)$$

$$V_a - V_b = 15.3 \text{ V}$$

**PART II: Conceptual Questions (each carries 1 point). Tick the best answer:**

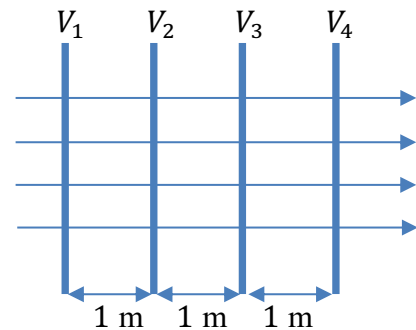
1. Two negative point charges,  $q_1 < 0$  and  $q_2 < 0$ , are placed on the  $x$ -axis as shown. We bring a positive charge,  $Q > 0$ , from infinity to point A. The electric potential at infinity is zero. The work of the electric force to move charge  $Q$  from infinity to point A is:

- a) zero.
- b) positive.
- c) negative.
- d) indetermined.



2. In a uniform electric field of magnitude  $E = 10 \text{ V/m}$ , the horizontal lines are the electric field lines while the vertical lines are the equipotential surfaces. If the potential of the first equipotential surface is  $V_1 = 95 \text{ V}$ , then

- a)  $V_2 = 95 \text{ V}, V_3 = 95 \text{ V}, V_4 = 95 \text{ V}$ .
- b)  $V_2 = 95 \text{ V}, V_3 = 85 \text{ V}, V_4 = 75 \text{ V}$ .
- c)  $V_2 = 85 \text{ V}, V_3 = 75 \text{ V}, V_4 = 65 \text{ V}$ .
- d)  $V_2 = 85 \text{ V}, V_3 = 65 \text{ V}, V_4 = 45 \text{ V}$ .



3. An air-filled parallel-plate capacitor is charged by a battery, then the battery is disconnected and the space between the plates of the capacitor is fully filled with a dielectric slab of constant  $K$ . Which statement is correct?

- a) The stored electric potential energy will decrease by a factor  $K$ .
- b) The stored electric potential energy will decrease by a factor  $2K$ .
- c) The stored electric potential energy will increase by a factor  $K$ .
- d) The stored electric potential energy will increase by a factor  $2K$ .



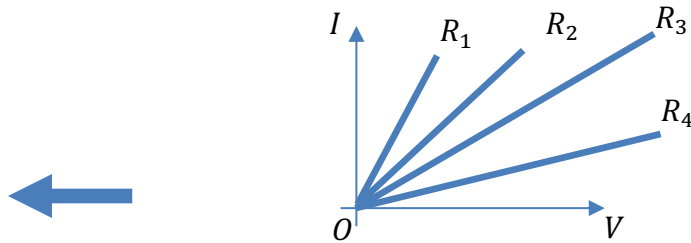
4. An air-filled parallel-plate capacitor is charged by a battery. While the battery remains connected, the separation between the plates of the capacitor is doubled. Which statement is correct?

- a) The electric energy density increases by a factor of 2.
- b) The electric energy density increases by a factor of 4.
- c) The electric energy density decreases by a factor of 2.
- d) The electric energy density decreases by a factor of 4.



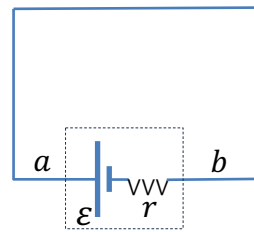
5. In the current-voltage diagram below, the characteristic curves of 4 resistors are given. Which relation is correct?

- a)  $R_1 > R_2 > R_3 > R_4$
- b)  $R_2 > R_1 > R_4 > R_3$
- c)  $R_3 > R_4 > R_2 > R_1$
- d)  $R_4 > R_3 > R_2 > R_1$



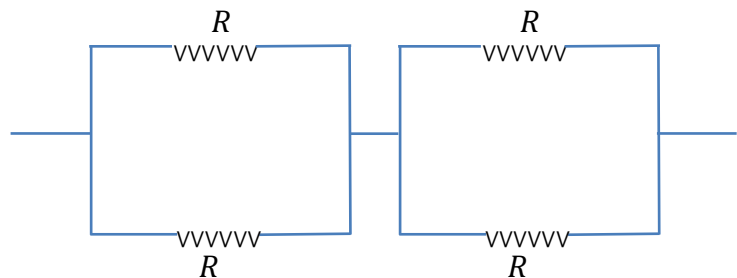
6. The current between points *a* and *b* in the electric circuit below is:

- a)  $I = \frac{\varepsilon}{R}$
- b)  $I = \frac{\varepsilon}{r}$
- c)  $I = \frac{\varepsilon}{R+r}$
- d)  $I = \frac{\varepsilon}{(R+r)^2}$



7. In the electric circuit below, there are 4 resistors of same resistance *R*. The equivalent resistance of this network is:

- a)  $R$ .
- b)  $2R$ .
- c)  $3R$ .
- d)  $4R$ .



8. Which statement is wrong?

- a) Kirchhoff's junction rule is based on the conservation of electric charge.
- b) Kirchhoff's loop rule is based on the statement that the electrostatic force is conservative.
- c) Kirchhoff's junction rule is about the sum of currents around any loop.
- d) Kirchhoff's loop rule is about the sum of potential differences around any loop.

