


**Second Midterm Examination
 Summer Semester 2022 – 2023**
July 15, 2023
Time: 12:00 – 1:30 PM

Name: Student No:

Section No: Serial No:

Instructors: Drs. Alaa Alfaiakawi, Afshin Hadipour, & Peter Lajko

Fundamental constants

$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2$	(Coulomb constant)
$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)$	(Permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$	(Permeability of free space)
$ e = 1.60 \times 10^{-19} \text{ C}$	(Elementary unit of charge)
$N_A = 6.02 \times 10^{23}$	(Avogadro's number)
$g = 9.8 \text{ m/s}^2$	(Acceleration due to gravity)
$m_e = 9.11 \times 10^{-31} \text{ kg}$	(Electron mass)
$m_p = 1.67 \times 10^{-27} \text{ kg}$	(Proton mass)

Prefixes of units

$m = 10^{-3}$	$\mu = 10^{-6}$	$n = 10^{-9}$	$p = 10^{-12}$
$k = 10^3$	$M = 10^6$	$G = 10^9$	$T = 10^{12}$

For use by Instructors only

Problems	1	2	3	4	5	6	7	8	Questions	Total
Marks										

Instructions to the Students:

1. Mobile or other electronic devices are **strictly prohibited** during the exam.
2. Programmable calculators, which can store equations, are not allowed.
3. **Cheating incidents will be processed according to the university rules.**

PART I: Solve the following problems. Show your solutions in detail.

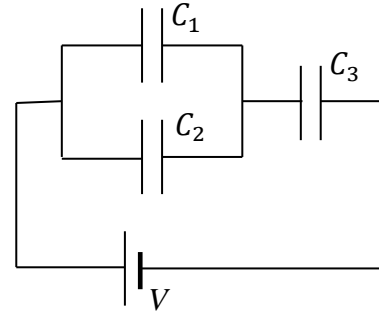
1. Three capacitors are connected to a potential difference as shown. If $C_1 = 3 \mu\text{F}$, $C_2 = 3 \mu\text{F}$, $C_3 = 4 \mu\text{F}$, and the energy stored in C_3 is $U_3 = 8 \mu\text{J}$, what is the potential difference V ? [5 points]

$$C_{12} = C_1 + C_2 = 3 \mu\text{F} + 3 \mu\text{F}$$

$$C_{eq} = \frac{C_3 \cdot C_{12}}{C_3 + C_{12}} = \frac{4 \mu\text{F} \times 6 \mu\text{F}}{(4 + 6) \mu\text{F}} = 2.4 \mu\text{F}$$

$$Q = Q_3 = \sqrt{2U_3 C_3} = 8 \mu\text{C}$$

$$V = \frac{Q}{C_{eq}} = 3.33 \text{ V}$$

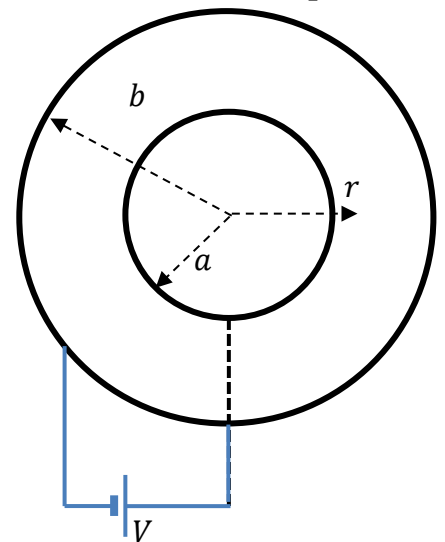


2. An air-filled spherical capacitor has inner radius $a = 4 \text{ cm}$, outer radius $b = 8 \text{ cm}$, and a potential difference of $V = 1200 \text{ V}$ is connected between its plates. Determine the energy density u_E at a distance $r = 5 \text{ cm}$ from its center. [4 points]

$$Q = CV = \frac{4\pi\epsilon_0 ab}{b-a} V = 10.7 \text{ nC}$$

$$E = \frac{kQ}{r^2} = 3.8 \times 10^4 \text{ N/C}$$

$$u_E = \frac{\epsilon_0}{2} E^2 = 6.4 \times 10^{-3} \text{ J/m}^3$$



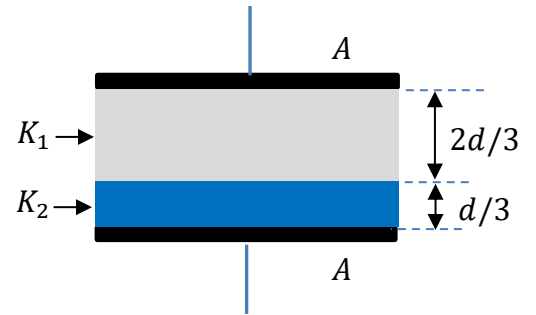
3. An air-filled parallel-plate capacitor with surface area A and plate separation d has capacitance $C_0 = 60 \mu\text{F}$. If the capacitor is filled with two dielectric materials with dielectric constants $K_1 = 6$ and $K_2 = 4$, as shown, calculate its capacitance. **[4 points]**

$$C_0 = \epsilon_0 \frac{A}{d}$$

$$C_1 = K_1 \epsilon_0 \frac{A}{2d/3} = 3K_1 \epsilon_0 \frac{A}{2d} = 9C_0$$

$$C_2 = K_2 \epsilon_0 \frac{A}{d/3} = 3K_2 \epsilon_0 \frac{A}{d} = 12C_0$$

$$C = \frac{C_1 C_2}{C_1 + C_2} = 308.6 \mu\text{F}$$



4. Cylindrical wires A and B are made of copper. Wire A has length L and radius r and wire B has length $2L$ and radius $2r$. When wire A is connected between a potential difference V , the power dissipation on it is $P_A = 24 \text{ W}$. If wire B is connected between the same potential difference V , calculate the power dissipation, P_B , on wire B . **[4 points]**

$$R_A = \frac{\rho L}{\pi r^2}; \quad R_B = \frac{\rho 2L}{\pi (2r)^2} = R_A/2$$

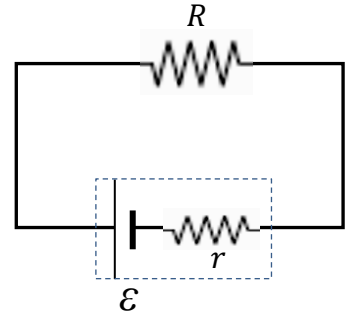
$$P_A = \frac{V^2}{R_A}$$

$$P_B = \frac{V^2}{R_B} = \frac{V^2}{R_A/2} = 48 \text{ W}$$

5. In the circuit shown below, $\varepsilon = 16.0 \text{ V}$, $R = 7.6 \Omega$ and the power dissipated on the external resistor R is $P_R = 30.4 \text{ W}$. Find the power dissipated on the internal resistance r of the battery. **[3 points]**

$$P_R = I^2 R \Rightarrow I = 2 \text{ A} = \frac{\varepsilon}{r+R} \Rightarrow r = 0.4 \Omega$$

$$P_r = I^2 r = 1.6 \text{ W}$$



6. In the circuit shown, the current through resistor R is 0.4 A . Calculate the power dissipation on the resistor R . **[5 points]**

Junction rule: $I_1 = I_2 + 0.4 \text{ A} \Rightarrow I_2 = I_1 - 0.4 \text{ A}$

Loop rule (clockwise) for the left loop:

$$18\text{V} - 4\Omega I_1 + 3\text{V} - 12\Omega(I_1 - 0.4 \text{ A}) = 0 \Rightarrow I_1 = 1.6 \text{ A}$$

Loop rule (clockwise) for the big loop gives

$$18\text{V} - 4\Omega I_1 - 0.4\text{A}R = 0 \rightarrow R = 29 \Omega$$

$$P = RI^2 = (29 \Omega) \times (0.4 \text{ A})^2 = 4.64 \text{ W}$$

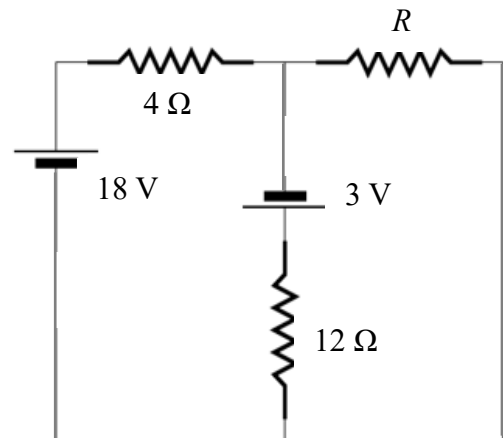
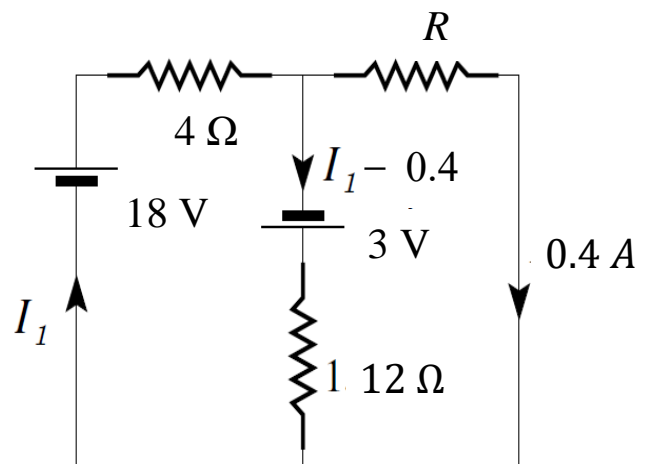
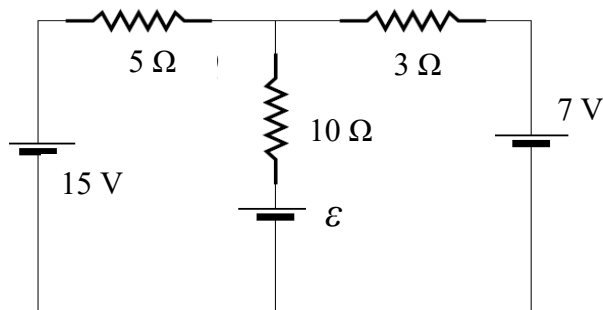


Figure for the solution:



7. The current is zero through the unknown *emf* source \mathcal{E} in the circuit shown. Calculate the value \mathcal{E} of the *emf* source. [4 points]



Loop rule for the outer loops (clockwise) gives:

$$15 - 5I - 3I - 7 = 0 \Rightarrow$$

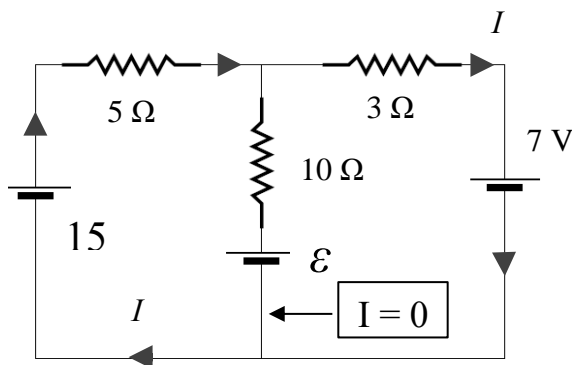
$$-8I = -15 + 7 \rightarrow I = 1 \text{ A}$$

Loop rule for the left loop (clockwise) gives:

$$15 - 5I - \mathcal{E} = 0$$

$$\mathcal{E} = 15 - 5I = 10 \text{ V}$$

Figure for the solution:



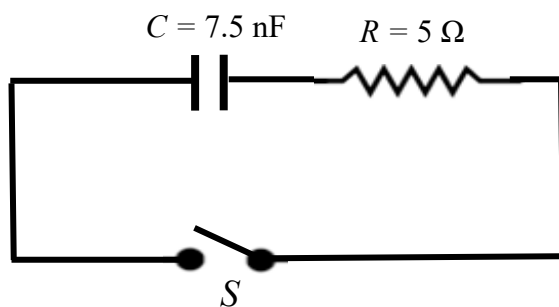
8. In the circuit below, the initial charge of the capacitor is $Q_0 = 65 \text{ nC}$. If the switch is closed at time $t = 0 \text{ s}$, calculate the power dissipated in the resistor at time $t = 1.8 \tau$. [3 points]

$$V = \frac{Q}{C} \quad \text{and} \quad q(t) = Q_0 e^{-\frac{t}{RC}} \rightarrow V = \frac{Q_0}{C} e^{-\frac{t}{RC}}$$

$$P = \frac{V^2}{R} = \frac{Q_0^2}{RC^2} e^{-\frac{2t}{RC}}$$

$$t = 1.8 RC \rightarrow P = \frac{Q_0^2}{RC^2} e^{-3.6}$$

$$P = 0.41 \text{ W}$$



PART II: Conceptual Questions (each carries 1 point). Tick the best answer:

1. An air-filled parallel-plate capacitor with plate separation d is charged by a potential difference V . While the potential difference remains connected, the plate separation is decreased to $d/2$. So, the energy stored on the capacitor

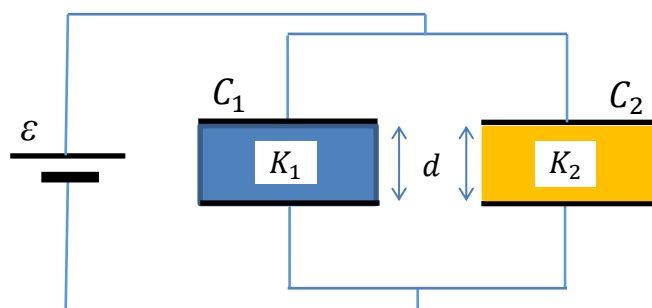
- a) does not change.
- b) becomes double. ←
- c) becomes half.
- d) becomes zero.

2. An air-filled spherical capacitor of capacitance C has inner radius a and outer radius b . A second spherical capacitor has inner radius $2a$ and outer radius $2b$ and fully filled with a dielectric material of $K = 2$. The second spherical capacitor has capacitance

- a) C .
- b) $2C$.
- c) $3C$.
- d) $4C$. ←

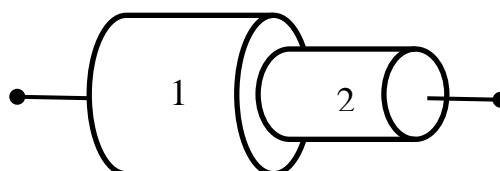
3. Two parallel-plate capacitors C_1 and C_2 , of identical separation d , are filled with materials of dielectric constant K_1 and K_2 , respectively, and are connected to potential source as shown. The magnitude of electric field is E_1 in C_1 , and it is E_2 in C_2 . Which relation is true for the electric fields?

- a) $\frac{E_1}{E_2} = \frac{K_1}{K_2}$.
- b) $\frac{E_1}{E_2} = \frac{K_2}{K_1}$.
- c) $\frac{E_1}{E_2} = 1$. ←
- d) $\frac{E_1}{E_2} = \left(\frac{K_1}{K_2}\right)^2$.

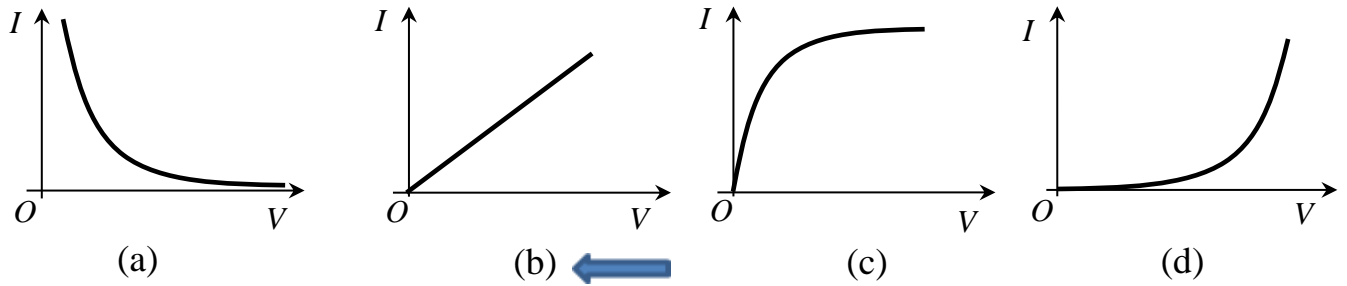


4. A potential difference is applied across the two ends of a conductor made of two cylinders of the same material with radii $r_1 = 2r_2$. If the magnitude of electric field is E_1 in wire 1, and it is E_2 in wire 2, which relation is correct?

- a) $E_1 = E_2/2$.
- b) $E_1 = E_2/4$. ←
- c) $E_1 = 2E_2$.
- d) $E_1 = 4E_2$.

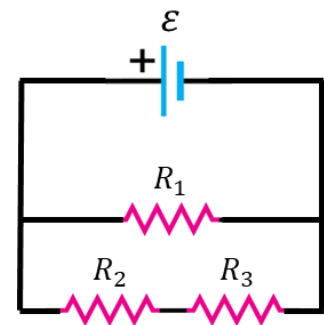


5. Which of the following figures does illustrate the electric current I in a resistor as a function of the potential difference V ?



6. In the circuit shown, $R_1 = R_2 = R_3$. Which of the statements below is correct for the currents in the resistors?

- a) $I_1 = I_2 = I_3$.
- b) $I_1 > I_2 > I_3$.
- c) $I_1 < I_2 < I_3$.
- d) $I_1 > I_2 = I_3$. ←



7. To simultaneously measure the current in and the voltage across the resistor in a circuit, we must connect

- a) ammeter and voltmeter in series connection with the resistor.
- b) ammeter in series and voltmeter in parallel connection with the resistor. ←
- c) ammeter and voltmeter in parallel connection with the resistor.
- d) ammeter in parallel and voltmeter in series connection with the resistor.

8. In the R - C circuit below, the switch is closed at time $t = 0$ s. After waiting a long time, the electric charge on each capacitor is

- a) $Q = \epsilon C$.
- b) $Q = 2\epsilon C$.
- c) $Q = \epsilon C/2$. ←
- d) $Q = \epsilon C[1 - e^{-t/RC}]$.

