Kuwait University **Physics Department**

جامعة الكويت
KUWAIT UNIVERSITY

Physics 102
First Midterm Exam

Summer Semester 2022/23
June 24, 2023

Time: 11:30 – 1:00 p.m.

Name…………………………………… Student No.…………………

Section No……………………………. Serial No………..……………..

Instructors: Drs. Al-Failakawi, Hadipour, & Lajko

Fundamental constants

$k = \dfrac{1}{4\pi\varepsilon_o} = 9.0\times10^{9}\ \mathrm{N.m^2/C^2}$	(Coulomb constant)
$\varepsilon_o = 8.85\times10^{-12}\ \mathrm{C^2/(N\cdot m^2)}$	(Permittivity of free space)
$\mu_0 = 4\pi\times10^{-7}\ \mathrm{T.m/A}$	(Permeability of free space)
$\lvert e\rvert = 1.60\times10^{-19}\ \mathrm{C}$	(Elementary unit of charge)
$N_A = 6.02\times10^{23}$	(Avogadro's number)
$g = 9.8\ \mathrm{m/s^2}$	(Acceleration due to gravity)
$m_e = 9.11\times10^{-31}\ \mathrm{kg}$	(Electron mass)
$m_p = 1.67\times10^{-27}\ \mathrm{kg}$	(Proton mass)

Prefixes of units: $m = 10^{-3}$, $k = 10^{3}$; $\mu = 10^{-6}$, $M = 10^{6}$; $n = 10^{-9}$, $G = 10^{9}$; $p = 10^{-12}$, $T = 10^{12}$

For use by Instructors only

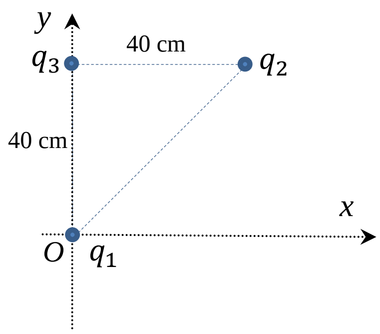
Prob.	1	2	3	4	5	6	7	8	Total
Marks									

Ques.	1	2	3	4	5	6	7	8	Total
Marks									

Important:
1. Mobiles or other electronic devices are **strictly prohibited** during the exam.
2. Programmable calculators, which can store equations, are not allowed.
3. **Cheating incidents will be processed according to the university rules.**

Part I. Solve the following problems. Show your solutions in detail.

1. Three point charges $q_1 = -4\ \mu\text{C}$, $q_2 = q_3 = 4\ \mu\text{C}$ are placed to the vertices of a right-angled triangle, as shown. Calculate the x and y components of the net electric force $\vec{F}_2$ acting on q_2. **[4 points]**

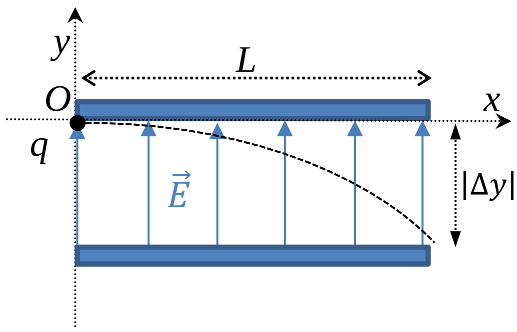


$$r_{12} = \sqrt{(0.4m)^2 + (0.4m)^2} = 0.566\ \text{m}$$

$$F_{2,x} = F_{13,x} + F_{23,x} = -k\frac{|q_1q_2|}{r_{12}^2}\cos(45^o)\ + k\frac{|q_2q_3|}{r_{23}^2} = 0.582\ \text{N}$$

$$F_{2,y} = F_{13,y}\ + F_{23,y} = -k\frac{|q_1q_3|}{r_{13}^2}\sin(45^o) = -0.318\ \text{N}$$

2. A particle of charge $q = -4\ \ \mu\text{C}$ and mass $m = 2\times10^{-6}$ kg moves with initial velocity $\vec{v} = 2\times10^2\left(\frac{\text{m}}{\text{s}}\right)\hat{\imath}$ into a region of uniform electric field $\vec{E} = 1200\ \left(\frac{\text{N}}{\text{C}}\right)\hat{\jmath}$. If the particle's path is bent as shown in the figure and $L = 2.0$ m, determine the displacement Δy. **[4 points]**



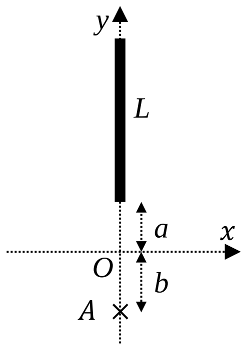
$$F = qE = ma \implies \quad a = \frac{qE}{m} = 2400\ \text{m/s}^2$$

$$t = \frac{L}{v} \implies t = 0.01$$

$$\Delta y = -a\frac{t^2}{2}$$

$$\Delta y = -0.12\ \text{m}$$

3. Charge $Q = 60$ nC is uniformly distributed along a rod of length $L = 3.0$ m fixed on the y-axis, as shown. Calculate the net electric field vector $\vec{E}_A$ at point A. Given $a = 0.3$ m and $b = 0.4$ m. **[5 points]**

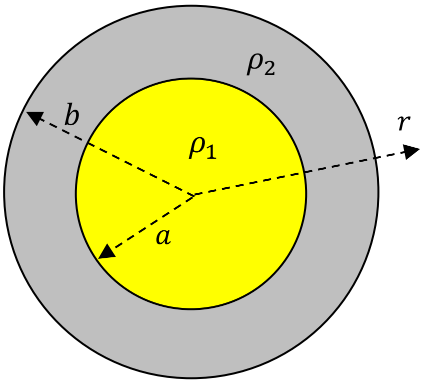


$d\vec{E}_A = \frac{kdQ}{r^2}(-\hat{j}) = \frac{k\lambda dy}{(y+b)^2}(-\hat{j})$; $\lambda = \frac{dQ}{dL} = \frac{Q}{L}$

$\vec{E}_A = \int_{source} d\vec{E}_A = -\int_a^{L+a} \frac{k\lambda dy}{(y+b)^2}\hat{j} = -k\lambda\left[-\frac{1}{y+b}\right]_a^{L+a}\hat{j} =$

$\vec{E}_A = -208.5\ \frac{\text{N}}{\text{C}}\ \hat{j}$

4. A sphere of radius $a = 4.0$ cm has uniform volume charge density $\rho_1 = +30.0$ nC/m^3 and a concentric spherical shell of inner radius a, outer radius $b = 8.0$ cm has uniform volume charge density $\rho_2 = -15.0$ nC/m^3. Find the net electric field (magnitude and direction (inward or outward)) at a distance $r = 12.0$ cm from the center point. **[4 points]**



Gauss's Law:

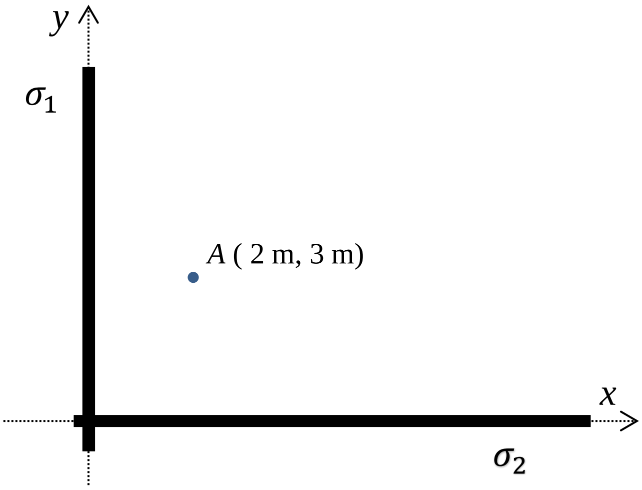
$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q_{encl}}{\varepsilon_0}$$

$$Q_{encl} = \rho_1\left(\frac{4}{3}\pi a^3\right) + \rho_2\left(\frac{4}{3}\pi(b^3 - a^3)\right)$$

$$E(4\pi r^2) = \frac{\rho_1\left(\frac{4}{3}\pi a^3\right)+\rho_2\left(\frac{4}{3}\pi(b^3-a^3)\right)}{\varepsilon_0} \Rightarrow E = -12.6\,\frac{\text{N}}{\text{C}}, \quad \text{inward}$$

5. A large sheet with uniform surface charge density $\sigma_1 = 8$ nC/m^2 is placed perpendicular to the x-axis and another large sheet with uniform surface charge density $\sigma_2 = 4$ nC/m^2 is perpendicular to the y-axis, as shown. Find the net electric field $\vec{E}$ at point A and its direction relative to the positive x-axis. **[4 points]**

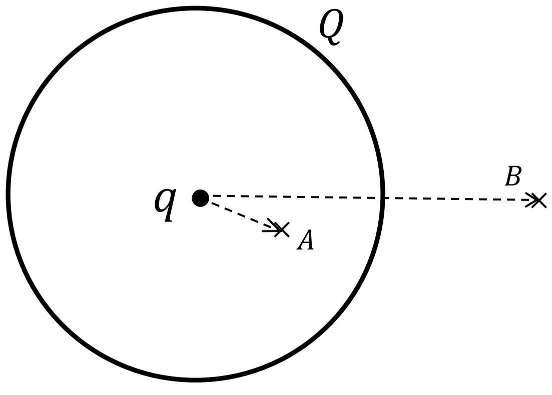


$$\vec{E}_{net} = \vec{E}_{1,A} + \vec{E}_{2,A} = \frac{\sigma_1}{2\varepsilon_0}\hat{\imath} + \frac{\sigma_2}{2\varepsilon_0}\hat{\jmath}$$

$$\vec{E}_{net} = 452\ \frac{N}{C}\hat{\imath} + 226\ \frac{N}{C}\hat{\jmath}$$

$$\theta = tan^{-1}\left[\frac{E_{net,y}}{E_{net,x}}\right] = 26.6^o$$

6. A spherical surface of radius $R = 0.5$ m has charge $Q = 10$ nC uniformly distributed on it and a point charge q is at its center. Take $V = 0$ at infinity. If the electric potential at point B is 0 V, what is the electric potential at point A? Given $r_A = 0.25$ m and $r_B = 1.0$ m. **[4 points]**

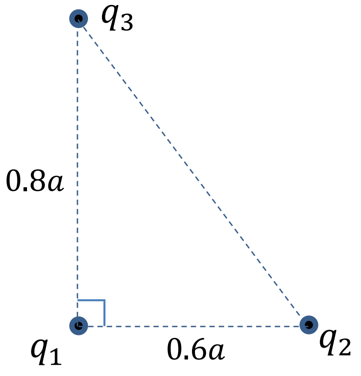


$$V_B = V_{qB} + V_{QB} = 0 = k\frac{q}{r_B} + k\frac{Q}{r_B} \Rightarrow$$

$$q = -Q = -10 \text{ nC}$$

$$V_A = V_{qA} + V_{QA} = k\frac{q}{r_A} + k\frac{Q}{R} = -180\ V$$

7. Three point charges are $q_1 = -2$ μC, $q_2 = q_3 = 4$ μC are fixed at vertices of a right-angled triangle, as shown. If the electric potential energy of the point charge q_1 is $U(q_1) = -2$ J, what is the total potential energy of the system. **[4 points]**

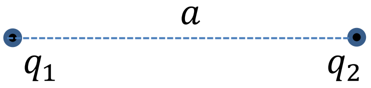


$$U(q_1) = k\frac{q_1q_2}{0.6a} + k\frac{q_1q_3}{0.8a} = -2\text{ J} \Rightarrow a = 0.105\, m$$

$$hypotenus = \sqrt{(0.8a)^2 + (0.6a)^2} = a$$

$$U_{tot} = k\frac{q_1q_2}{0.6a} + k\frac{q_1q_3}{0.8a} + k\frac{q_2q_3}{a} = -0.63\text{ J}$$

8. Two point charges, $q_1 = q_2 = 2$ μC, with identical masses, $m_1 = m_2 = 4$ g, are released simultaneously from rest at a distance $a = 0.5$ m, as shown. What is the speed of q_1 when it is at a distance b =1.5 m from q_2? **[3 points]**



The mechanical energy conservation:

$$E_{in} = E_{fin} \Rightarrow U_{in} = U_{fin} + K_{fin}$$

$$k\frac{q_1q_2}{a} = k\frac{q_1q_2}{b} + 2\frac{mv^2}{2} \Rightarrow kq_1q_2\left(\frac{1}{a} - \frac{1}{b}\right) = mv^2$$

$$v = \sqrt{\frac{kq_1q_2}{m}(\frac{1}{a} - \frac{1}{b})} = 3.46\text{ m/s}$$

Part II. Multiple choice questions (each carries 1 point). Tick the best answer:

1. Three identical metal spheres A, B, and C are initially uncharged and separated. Then sphere A is charged with 40 μC and spheres A and B are touched together and then separated. Sphere C is then touched to sphere B and separated from it. What is the final charge of sphere C?

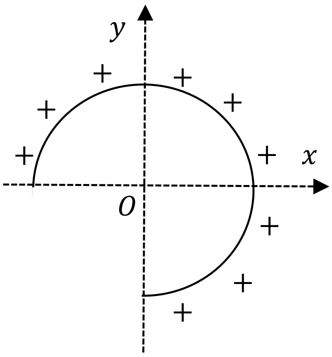
   a) 40 μC.

   b) 20 μC.

   c) 10 μC.

   d) 0.

2. The arc in the figure is uniformly charged as shown. The direction of the net electric field at the origin is



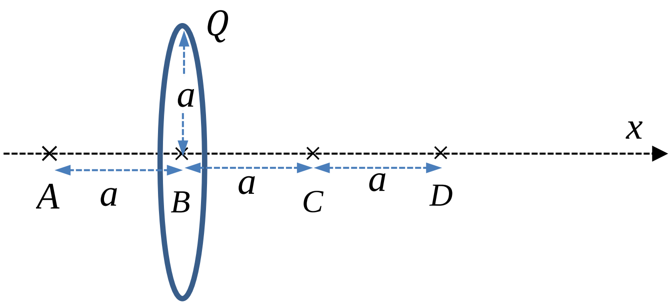
   a) $+\hat{\imath}+\hat{\jmath}$.

   b) $-\hat{\imath}-\hat{\jmath}$.

   c) $+\hat{\imath}-\hat{\jmath}$.

   d) $-\hat{\imath}+\hat{\jmath}$.

3. A charged ring of radius a is placed along the x-axis as shown. At which point is the magnitude of the electric field smallest?



   a) A.

   b) B.

   c) C.

   d) D.

4. If E is the magnitude of electric field at a distance $r=\frac{R}{4}$ from the center of a uniformly charged non-conducting sphere of radius R, the magnitude of electric field at a distance R from the center of the sphere is

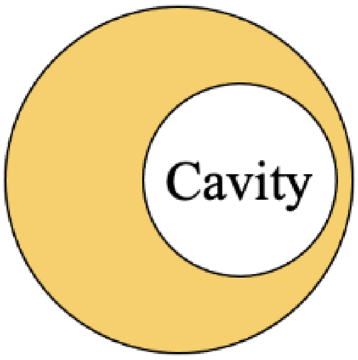
   a) 0.

   b) E.

   c) $2E$.

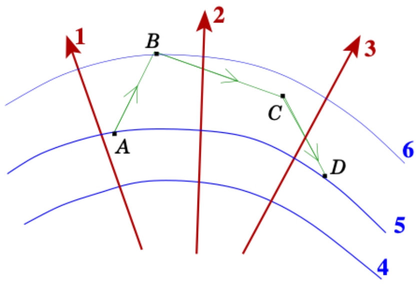
   d) $4E$.

5. A solid spherical conductor has a spherical cavity, as shown. If there is a net positive charge on the outer surface of the conductor, the electric field in the cavity



a) points generally from the center of the conductor towards the outer surface of the cavity.
b) points generally from the outer surface of the conductor toward the center of the cavity.
c) is zero.
d) points from the center of the cavity toward the center of the conductor.

6. The straight lines 1, 2, and 3 represent the electric field lines, and the curved lines marked 4, 5, and 6 represent the equipotential surfaces, as shown. A charge q follows the path $A \rightarrow B \rightarrow C \rightarrow D$. The work done on the charge by the electric field is

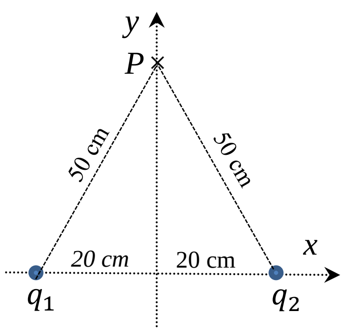


a) $W_E > 0$, if q is positive
b) $W_E < 0$, if q is positive
c) $W_E < 0$, if q is negative.
d) $W_E = 0$.

7. Which of the following is the unit of the energy?

a) V·C.
b) N·C/m.
c) J/C.
d) V/C.

8. Two point charges are placed along the x-axis, as shown. If the electric potential is positive at point P, relative to $V = 0$ at infinity, which statement is correct for the charges?



a) $q_1 = q_2$.
b) $q_1 = -q_2$.
c) $q_1 + q_2 > 0$.
d) $q_1 + q_2 < 0$.