



Final Examination
Summer Semester 2023 – 2024

July 29, 2024

Time: 5:00 – 7:00 PM

Name: Student No: Sec. No: Serial No:

Instructors: Drs. Alaa Alfailakawi, Peter Lajko, and Elias Vagenas

Fundamental constants

$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2$	(Coulomb constant)
$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)$	(Permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A}$	(Permeability of free space)
$ e = 1.60 \times 10^{-19} \text{ C}$	(Elementary unit of charge)
$N_A = 6.02 \times 10^{23}$	(Avogadro's number)
$g = 9.8 \text{ m/s}^2$	(Acceleration due to gravity)
$m_e = 9.11 \times 10^{-31} \text{ kg}$	(Electron mass)
$m_p = 1.67 \times 10^{-27} \text{ kg}$	(Proton mass)

Prefixes of units

$m = 10^{-3}$	$\mu = 10^{-6}$	$n = 10^{-9}$	$p = 10^{-12}$
$k = 10^3$	$M = 10^6$	$G = 10^9$	$T = 10^{12}$

For use by Instructors only

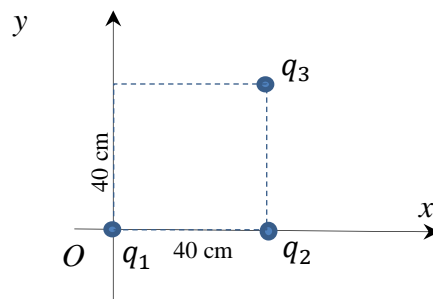
Problems	1	2	3	4	5	6	7	8	9	10	Questions	Total
Marks												

Instructions to the Students:

1. Mobile or other electronic devices are **strictly prohibited** during the exam.
2. Programmable calculators, which can store equations, are not allowed.
3. Cheating incidents will be processed according to the university rules.

PART I: Solve the following problems. Show your solutions in detail.

1. Three point charges, $q_1 = q_2 = q_3 = 2 \mu\text{C}$, are placed on the vertices of a square, as shown. Calculate the x and y components of the net electric force, \vec{F}_1 , acting on q_1 . [5 points]



$$r_{13} = \sqrt{(0.4 \text{ m})^2 + (0.4 \text{ m})^2} = 0.5657 \text{ m}$$

$$F_{31,x} = -k \frac{|q_3 q_1|}{r_{13}^2} \cos(45^\circ) = -0.08 \text{ N};$$

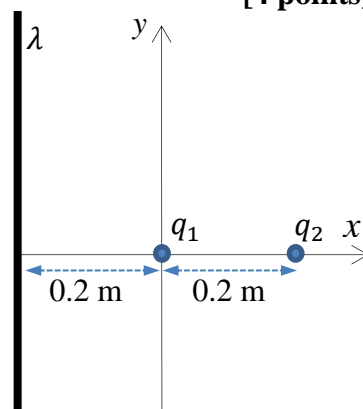
$$F_{31,y} = -k \frac{|q_3 q_1|}{r_{13}^2} \sin(45^\circ) = -0.08 \text{ N};$$

$$F_{21,x} = -k \frac{|q_1 q_2|}{r_{21}^2} = -0.225 \text{ N}; F_{21,y} = 0$$

$$F_{1,x} = F_{21,x} + F_{31,x} = -0.305 \text{ N}$$

$$F_{1,y} = F_{21,y} + F_{31,y} = -0.08 \text{ N}$$

2. Two equal point charges $q_1 = q_2 = 2 \mu\text{C}$ are placed on the x -axis and a very long uniformly charged line, with uniform linear charge density, λ , is placed perpendicular to the x -axis, as shown. If the net force acting on the point charge q_2 is zero, find the value of λ . [4 points]



$$\vec{E}_\lambda = \frac{\lambda}{2\pi\epsilon_0 0.4 \text{ m}} \hat{i}$$

$$\vec{E}_{q_1} = \frac{kq_1}{(0.2 \text{ m})^2} \hat{i}$$

$$\vec{F}_{net} = q_2(\vec{E}_1 + \vec{E}_2) = 0 \Rightarrow$$

$$\lambda = -\frac{q_2 0.4 \text{ m}}{2(0.2 \text{ m})^2} = -10.0 \mu\text{C}/\text{m}$$

3. A spherical shell of inner radius $a = 10$ cm and outer radius $b = 20$ cm has uniform volume charge density $\rho = -300$ nC/m³. Determine the magnitude and direction of the net electric field at distance of 15 cm from the center.

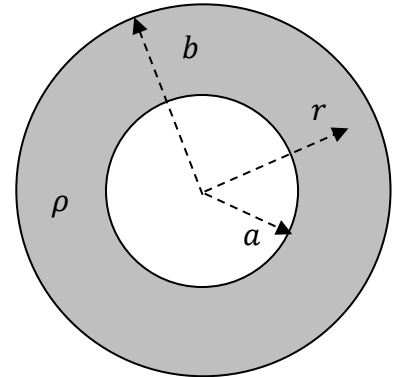
[3 points]

Gauss's Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{encl} = \rho \left(\frac{4}{3} \pi (r^3 - a^3) \right)$$

$$E = \frac{Q_{enc}}{4\pi r^2 \epsilon_0} = -1193 \text{ N/C, inward}$$

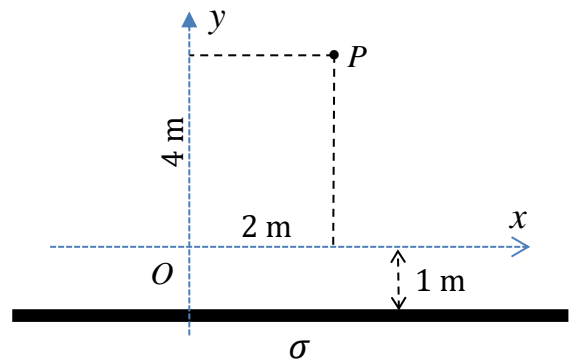


4. A very large sheet with uniform surface charge density, $\sigma = 70.8$ $\mu\text{C}/\text{m}^2$, is placed perpendicular to the y-axis, as shown. If a point charge, $q = 4$ μC , is moved from point P to O, calculate the work done by the electric field.

[3 points]

$$W_E = \int_P^O \vec{F} \cdot d\vec{l} = \vec{F} \cdot \Delta\vec{L}$$

$$\vec{F} \cdot \Delta\vec{L} = -\frac{\sigma q}{2\epsilon_0} \Delta y = -64 \text{ J}$$

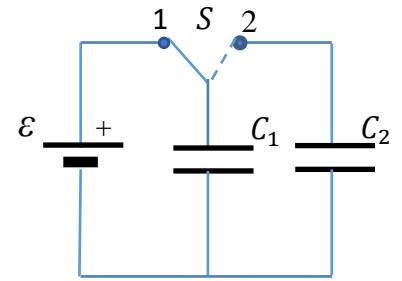


5. A capacitor, $C_1 = 20 \mu\text{F}$, is charged by the battery, $\mathcal{E} = 60 \text{ V}$, while the switch S is position 1, as shown. Then the switch is moved to position 2 so that capacitor C_1 is connected to the uncharged capacitor $C_2 = 40 \mu\text{F}$. Calculate the final energy stored in capacitor C_2 .

[3 points]

$$V^{fin} = \frac{Q_0}{C_{eq}} = \frac{C_1}{C_1 + C_2} \mathcal{E} \Rightarrow V^{fin} = \frac{\mathcal{E}}{3} = 20 \text{ V}$$

$$U_2^{fin} = \frac{C_2 (V^{fin})^2}{2} = 8 \text{ mJ}$$



6. A cylindrical wire of resistivity $\rho = 1.72 \times 10^{-8} \Omega \cdot \text{m}$, length L , and diameter $d = 0.4 \text{ mm}$ is connected to a potential difference of $V = 60 \text{ V}$. If the power dissipation on the wire is $P = 144 \text{ W}$, calculate the electric field in the wire.

[3

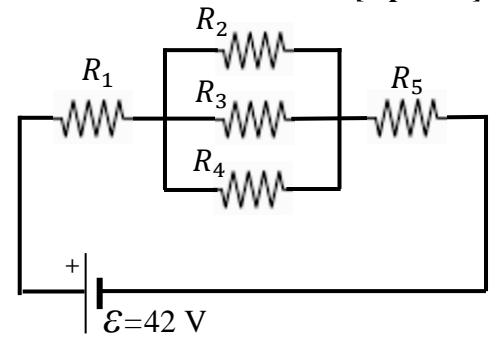
points]

$$P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = 25 \Omega$$

$$R = \rho \frac{L}{A} \Rightarrow L = \frac{R \pi r^2}{\rho} = 183 \text{ m}$$

$$E = \frac{V}{L} = 0.33 \text{ N/C}$$

7. Five identical resistors, $R_1 = R_2 = R_3 = R_4 = R_5 = 6 \Omega$, are connected into circuit, as shown. Find the current on resistor R_4 . **[4 points]**



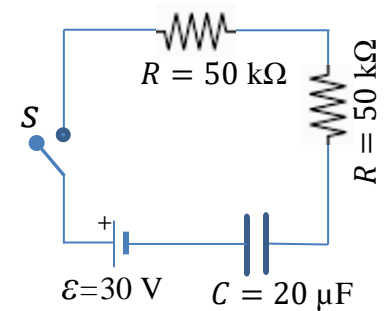
$$R_{234} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = 2 \Omega$$

$$R_{eq} = R_1 + R_{234} + R_5 = 14 \Omega$$

$$I_{\mathcal{E}} = \frac{\mathcal{E}}{R_{eq}} = 3 \text{ A}$$

$$R_4 I_{R_4} = R_{234} I_{R_{234}} \Rightarrow I_{R_4} = 1 \text{ A}$$

8. In the circuit below, the capacitor is initially uncharged and the switch is closed at time $t = 0$ s. Calculate the time t_1 at which the charge of the capacitor is 1/4 of its final value. **[5 points]**



$$\tau = R_{eq} C = 2RC = 2 \text{ s}$$

$$q(t) = \varepsilon C \left[1 - e^{-\frac{t}{R_{eq} C}} \right] \Rightarrow Q_{fin} = \varepsilon C$$

$$\frac{Q_{fin}}{4} = q(t_1) = \varepsilon C \left[1 - e^{-\frac{t_1}{R_{eq} C}} \right]$$

$$\frac{1}{4} = 1 - e^{-\frac{t_1}{R_{eq} C}} \Rightarrow \frac{3}{4} = e^{-\frac{t_1}{R_{eq} C}}$$

$$t_1 = -R_{eq} C \ln\left(\frac{3}{4}\right) = 0.575 \text{ s}$$

9. A point charge $q = 0.6 \text{ mC}$ moves momentarily with velocity $\vec{v} = (8000 \frac{\text{m}}{\text{s}}) \hat{j}$ in a region of uniform magnetic field $\vec{B} = (3 \text{ T}) \hat{i} + (4 \text{ T}) \hat{k}$. Calculate the magnitude of magnetic force \vec{F} acting on the point charge. **[4 points]**

$$\vec{F} = q \vec{v} \times \vec{B} = 0.6 \text{ mC} \cdot (8000 \frac{\text{m}}{\text{s}}) \hat{j} \times ((3 \text{ T}) \hat{i} + (4 \text{ T}) \hat{k})$$

$$\vec{F} = (19.2 \text{ N}) \hat{i} - (14.4 \text{ N}) \hat{k}$$

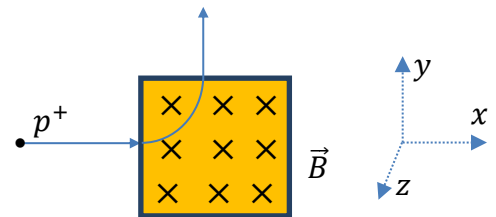
$$F = 24.0 \text{ N}$$

10. A proton moves into a region of uniform magnetic field with a velocity $\vec{v} = (1500 \frac{\text{m}}{\text{s}}) \hat{i}$ and leaves it with a velocity $\vec{v} = (1500 \frac{\text{m}}{\text{s}}) \hat{j}$. If the proton travels a distance $s = 0.25 \text{ m}$ in the magnetic field, find the magnitude of the magnetic field. **[3 points]**

The path of the proton is a quarter circle.

$$s = \frac{2\pi R}{4} \Rightarrow R = \frac{4s}{2\pi} = 0.159 \text{ m}$$

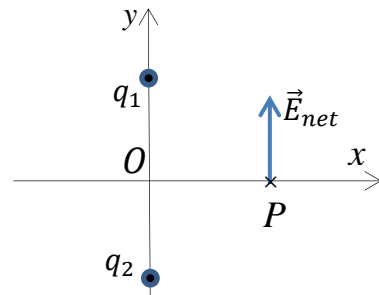
$$R = \frac{mv}{qB} \Rightarrow B = \frac{mv}{qR} = 9.84 \times 10^{-5} \text{ T}$$



PART II: Conceptual Questions (each carries 1 point). Tick the best answer:

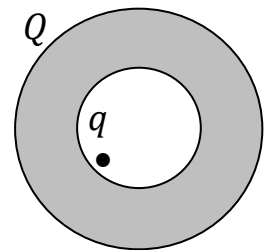
1. Two point charges q_1 and q_2 are fixed on the y -axis as shown. If the net electric field \vec{E}_{net} at point P is along the y -axis, which statement can be correct for the charges?

- a) $q_1 = q_2$.
- b) $q_1 < 0$ and $q_2 < 0$.
- c) $q_1 > 0$ and $q_2 > 0$.
- d) $q_1 < 0$ and $q_2 > 0$. ←



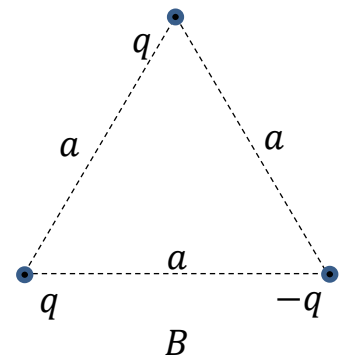
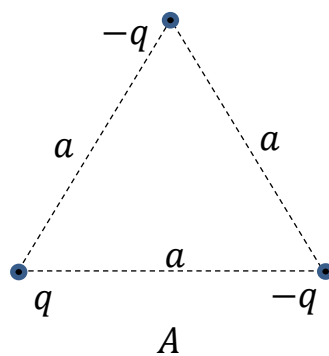
2. A conducting spherical shell has a net charge Q and a point charge q is fixed off-center of its cavity, as shown. Which statement is correct? The surface charge density is

- a) uniform on the inner surface and non-uniform on the outer surface.
- b) uniform on the inner surface and uniform on the outer surface.
- c) non-uniform on the inner surface and uniform on the outer surface. ←
- d) non-uniform on the inner surface and non-uniform on the outer surface.



3. Systems A and B are both made of 3 point charges, as shown. Which statement is true for the total potential energies of these systems, U_A , and U_B , respectively?

- a) $U_A = -U_B$.
- b) $U_A = U_B$. ←
- c) $U_A = 2U_B$.
- d) $U_A = 3U_B$.



4. If two identical capacitors are connected in series, their equivalent capacitance is C_{ser} . If the same capacitors are connected in parallel, their equivalent capacitance is C_{par} . The ratio of C_{ser}/C_{par} is

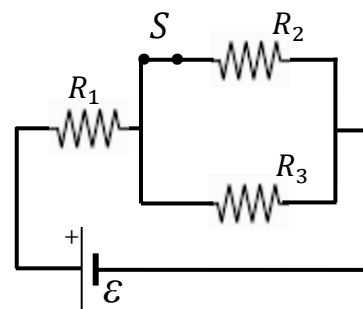
- a) $1/2$.
- b) $1/4$. ←

- c) 1.
 d) 2.
 5. A cylindrical rod has resistance R . If the length and the diameter of the rod are both doubled, the rod has a resistance

- a) $R/2$. ←
 b) $R/4$.
 c) $2R$.
 d) $4R$.

6. In the circuit shown, $R_1 = R_2 = R_3$. If the switch, S , is opened, the current supplied by the emf device

- a) decreases. ←
 b) increases.
 c) remains the same.
 d) drops to zero.



7. The magnetic force acting a charged particle can never do work because the force is

- a) parallel with the velocity of the particle.
 b) perpendicular to the velocity of the particle. ←
 c) parallel with the magnetic field.
 d) opposite to the magnetic field.

8. Three straight wires A , B , and C , with identical lengths carrying identical currents I , are placed into a uniform magnetic field, as shown. The magnitude of magnetic force is

- a) smallest for wire A .
 b) largest for wire B .
 c) largest for wire C .
 d) smallest for wire C . ←

