

For Instructors use only

Grades:

Important:

- 1. Answer all questions and problems (No solution = no points).
- 2. Full mark = 40 points as arranged in the above table.
- 3. **Give your final answer in the correct units.**
- 4. Assume $g = 10 \text{ m/s}^2$.
- 5. Mobiles are **strictly prohibited** during the exam.
- 6. Programmable calculators, which can store equations, are not allowed.
- 7. **Cheating incidents will be processed according to the university rules.**

GOOD LUCK

Part I: Short Problems (3 points each)

SP1. An object has a position vector given by $\vec{r}(t) = [(2 + t^3)\hat{i} + (3 - 4t^2)\hat{j}] m$, where t is measured in seconds. Find the magnitude of the object's acceleration at $t = 2$ s.

$$
\vec{v} = \frac{d\vec{r}}{dt} = (3t^2\hat{i} - 8t\hat{j}) \, m/s
$$
\n
$$
\vec{a} = \frac{d\vec{v}}{dt} = (6t\hat{i} - 8\hat{j}) \, m/s^2
$$
\n
$$
\vec{a}(2s) = (6(2)\hat{i} - 8\hat{j}) \, m/s^2 = (12\hat{i} - 8\hat{j}) \, m/s^2
$$
\n
$$
|\vec{a}(2s)| = \sqrt{12^2 + 8^2} = 14.4 \, m/s^2
$$

SP2. A 15 kg block is pushed to the right along a horizontal **frictionless** surface by a constant force $F = 120 N$, as shown. **Draw the free body diagram of the block and find its acceleration.**

SP3. A 2 kg block is pulled across **a rough** inclined surface by a force $F = 18 N$, as shown. The block **moves up** the incline with **constant speed**. **Find the coefficient of kinetic friction between the block and the surface.**

$$
F - mg \sin \theta - f_k = 0
$$

$$
F - mg \sin \theta - \mu_k mg \cos 37^\circ = 0
$$

$$
\mu_k = \frac{F - mg \sin \theta}{mg \cos 37^\circ} = 0.37
$$

SP4. A 2 kg block **rests** on a frictionless surface. A 50 g bullet moving at 400 m/s strikes the block and **emerges from it** with a speed of 250 m/s , as shown. **Find the speed of the block after the collision.**

SP5. Two forces, $F_1 = 30 N$ and $F_2 = 40 N$, are applied on **a massless rod** with a length of $L = 3 m$, as shown. Calculate the net torque about point $\boldsymbol{0}$ due to these forces.

2

SP6. A block of mass *m* slides **from rest** on a frictionless loop-the-loop track, as shown. What is the **minimum** release height (h) required for the block to **stay in contact with the surface at the top of the circular portion of the track**?

at the top of the circular portion

$$
mg + n = m\frac{v^2}{r}
$$

$$
n = 0 \Rightarrow v = \sqrt{rg} = \sqrt{20} \, m/s
$$

$$
mg(h-4) = \frac{1}{2}mv^2 \Rightarrow h = \frac{1}{2}\frac{v^2}{g} + 4 = 5 \, m
$$

SP7. A 2 kg box **rests** on **a frictionless inclined surface** and attached to a light rope that passes over a frictionless pulley ($R = 0.1$ m , $I = 0.01$ $kg \cdot m^2$), as shown. The rope is pulled from its lower end by a constant force $(F = 25 N)$. If the block **starts** from rest at point A, find its speed at point B.

$$
E_f - E_i = W_F
$$

\n
$$
mgh + \frac{1}{2}mv^2 + \frac{1}{2}I(\frac{v}{R})^2 - 0 = F(d)
$$

\n
$$
2(10)(3\sin(30^\circ)) + \frac{1}{2}(2)v^2 + \frac{1}{2}(\frac{0.01}{0.1^2})v^2 = 25(3)
$$

\n
$$
v = 5.5 \, m/s
$$

Part II: Long Problems (5 points each)

LP1. Two small balls are connected by **a very light** rigid rod, as shown. The system rotates around the $x - axis.$

a) Find the moment of inertia of the system about the axis of rotation.

b) The system starts to rotate **from rest** with a constant angular acceleration of 20 rad/s^2 . Find the rotational kinetic energy of the system at $t = 3$ s.

$$
\omega_f = \omega_i + \alpha t
$$

= 0 + 20(3) = 60 rad/s

$$
K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.03)(60)^2 = 54 J
$$

c) Find the tangential speed of m_1 at $t = 3$ s.

$$
v = R\omega = 0.2(60) = 12 \, m/s
$$

LP2. A 2 kg block **rests** on **a frictionless horizontal** surface is attached to a spring of force constant $k = 40 N/m$, as shown. **The spring is relaxed at point A**, you move the block by pulling with a constant horizontal force $F = 20 N$.

(a) Find the **total work** done on the block as it moves **from point A to point B.**

$$
\sum W = W_F + W_{F_S} = F(d) + \frac{1}{2}k(x_i^2 - x_f^2)
$$

= 20(0.25) + $\frac{1}{2}$ (40)(0²_i - 0.25²) = 3.75 J

(b) Find the block's speed when it reaches point B.

$$
\sum W = \Delta K
$$

3.75 = $\frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}(2)(v_f^2 - 0^2) \Rightarrow v_f = 1.9 \, m/s$

(c) If the surface was rough, then the **total work** done on the block as it moves **from point A to point B** will be

 *** less than the value obtained in part a.**

- *** greater than the value obtained in part a.**
- *** equal to the value obtained in part a.**

LP3. block 1 ($m_1 = 3 kg$) moves on **a horizontal frictionless surface** with an initial velocity $\vec{v}_{1_i} = (5\hat{i} - 4\hat{j})$ *m/s*. It collides with block 2 ($m_2 = 2$ kg), which **is initially at rest**. After the collision, the velocity of m_1 is $\vec{v}_{1_f} = (4\hat{i} - 2\hat{j}) m/s$.

(a) Find the velocity of block 2 after the collision in unit vector notation.

$$
m_1 \vec{v}_{1_i} + m_2 \vec{v}_{2_i} = m_1 \vec{v}_{1_f} + m_2 \vec{v}_{2_f}
$$

3(5 \hat{i} - 4 \hat{j}) + 0 = 3(4 \hat{i} - 2 \hat{j}) + 2 \vec{v}_{2_f}

$$
\vec{v}_{2_f} = \frac{(15\hat{i} - 12\hat{j}) - (12\hat{i} - 6\hat{j})}{2} = (1.5\hat{i} - 3\hat{j}) m/s
$$

(b) If the collision lasts for 0.1s, find the average net force exerted **on block** 1 during the collision in unit vector notation.

$$
\left(\sum F\right)_{av_{on block 1}} = \frac{\Delta \vec{p}_1}{\Delta t} = \frac{m_1 \left(\vec{v}_{1_f} - \vec{v}_{1_i}\right)}{\Delta t} = \frac{3((4\hat{i} - 2\hat{j}) - (5\hat{i} - 4\hat{j}))}{0.1} = (-30\hat{i} + 60\hat{j}) N
$$

(c) Find the change in kinetic energy of the two blocks system due to the collision.

$$
v_{1_i}^2 = 5^2 + 4^2 = 41 (m/s)^2
$$

\n
$$
v_{1_f}^2 = 4^2 + 2^2 = 20 (m/s)^2
$$

\n
$$
v_{2_f}^2 = 1.5^2 + 3^2 = 11.25 (m/s)^2
$$

\n
$$
\Delta K = \left(\frac{1}{2}m_1v_{1_f}^2 + \frac{1}{2}m_2v_{2_f}^2\right) - \left(\frac{1}{2}m_1v_{1_i}^2 + \frac{1}{2}m_2v_{2_i}^2\right)
$$

\n
$$
= \left(\frac{1}{2}(3)(20) + \frac{1}{2}(2)(11.25)\right) - \left(\frac{1}{2}(3)(41) + 0\right) = -20.25 J
$$

Part III: Questions (Choose the correct answer, one point each)

Q1. A box starts **from rest at** $t = 0$ **s** and moves along the positive x-axis with **constant acceleration**. If the speed of the box at $t = 1$ s is v, then its speed at $t = 3$ s is:

*
$$
v
$$
 * $2v$ * $2v$ * $9v$

Q2. When the bob (*mass* = m) of a simple pendulum moves from point A to point B, as shown, the work done on the bob by the **tension** is equal to

Q3. An object is **initially moving to the right** along $+x$ -axis with velocity v_{xi} . A net force is exerted on the object giving it **an impulse towards the left**. After the impulse, which of following statements is always true about the final velocity of the object v_{xf} :

- v_{xf} is negative.
- v_{xf} is zero.
- v_{xf} is positive.
- \bigodot All the above are possible.

Q4. An object is moving in a circular path counterclockwise and **speeding up at a constant rate**. The object passes point A and then point B. Which figure represents the total acceleration of the object at these two points?

