

# For Instructors use only

### Grades:

#	SP1	SP2	SP3	SP4	SP5	SP6	SP7	LP1	LP2	LP3	Q1	Q2 (Q3 Q	Total
	3	3	3	3	3	3	3	5	5	5	1	1 1	40
Pts												1	

### Important:

- 1. Answer all questions and problems (No solution = no points).
- 2. Full mark = 40 points as arranged in the above table.
- 3. Give your final answer in the correct units.
- 4. Assume  $g = 10 \text{ m/s}^2$ .
- 5. Mobiles are **<u>strictly prohibited</u>** during the exam.
- 6. Programmable calculators, which can store equations, are not allowed.
- 7. Cheating incidents will be processed according to the university rules.

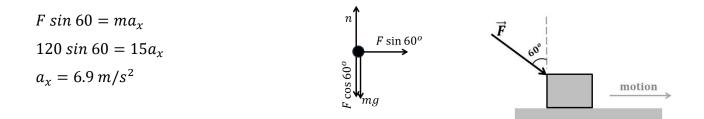
## GOOD LUCK

#### Part I: Short Problems (3 points each)

**SP1.** An object has a position vector given by  $\vec{r}(t) = [(2 + t^3)\hat{i} + (3 - 4t^2)\hat{j}]m$ , where t is measured in seconds. Find the <u>magnitude</u> of the object's acceleration at t = 2 s.

$$\vec{v} = \frac{d\vec{r}}{dt} = (3t^2\hat{\iota} - 8t\hat{j}) \ m/s$$
$$\vec{a} = \frac{d\vec{v}}{dt} = (6t\hat{\iota} - 8\hat{j}) \ m/s^2$$
$$\vec{a}(2s) = (6(2)\hat{\iota} - 8\hat{j}) \ m/s^2 = (12\hat{\iota} - 8\hat{j}) \ m/s^2$$
$$|\vec{a}(2s)| = \sqrt{12^2 + 8^2} = 14.4 \ m/s^2$$

SP2. A 15 kg block is pushed to the right along a horizontal frictionless surface by a constant force F = 120 N, as shown. Draw the free body diagram of the block and find its acceleration.

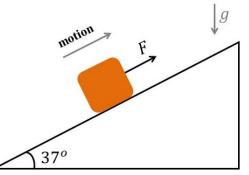


**SP3.** A 2 kg block is pulled across a rough inclined surface by a force F = 18 N, as shown. The block moves up the incline with <u>constant speed</u>. Find the coefficient of kinetic friction between the block and the surface.

$$F - mg\sin\theta - f_k = 0$$
  

$$F - mg\sin\theta - \mu_k mg\cos 37^o = 0$$
  

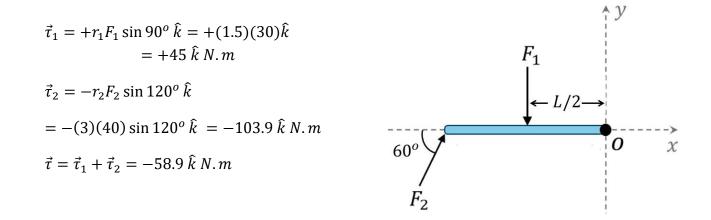
$$\mu_k = \frac{F - mg\sin\theta}{mg\cos 37^o} = 0.37$$



SP4. A 2 kg block rests on a frictionless surface. A 50 g bullet moving at 400 m/s strikes the block and emerges from it with a speed of 250 m/s, as shown. Find the speed of the block after the collision.

	Before the collision	After the collision
$m_1 v_{1x_i} + m_2 v_{2x_i} = m_1 v_{1x_f} + m_2 v_{2x_f}$		$v_{2x_f}$
$0.05 (400) + 0 = 0.05 (250) + 2v_{2x_f}$	400 m/s	250 m/s
$\frac{0.05(400) - 0.05(250)}{0.05(250)}$		
$v_{2x_f} =2$		
= 3.75 m/s		

**SP5.** Two forces,  $F_1 = 30 N$  and  $F_2 = 40 N$ , are applied on <u>a massless</u> rod with a length of L = 3 m, as shown. Calculate the net torque about point *O* due to these forces.

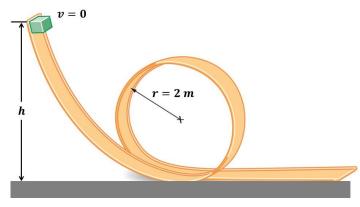


**SP6.** A block of mass *m* slides **from rest** on a frictionless loop-the-loop track, as shown. What is the minimum release height (*h*) required for the block to <u>stay in contact with the surface</u> at the top of the circular portion of the track?

at the top of the circular portion

$$mg + n = m\frac{v^2}{r}$$
$$n = 0 \Rightarrow v = \sqrt{rg} = \sqrt{20} m/s$$

$$mg(h-4) = \frac{1}{2}mv^2 \Rightarrow h = \frac{1}{2}\frac{v^2}{g} + 4 = 5 m$$



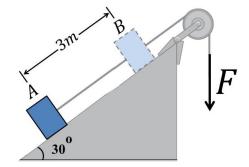
**SP7.** A 2 kg box rests on a frictionless inclined surface and attached to a light rope that passes over a frictionless pulley (R = 0.1 m,  $I = 0.01 kg \cdot m^2$ ), as shown. The rope is pulled from its lower end by a constant force (F = 25 N). If the block starts from rest at point A, find its speed at point B.

$$E_{f} - E_{i} = W_{F}$$

$$mgh + \frac{1}{2}mv^{2} + \frac{1}{2}I(\frac{v}{R})^{2} - 0 = F(d)$$

$$2(10)(3\sin(30^{o})) + \frac{1}{2}(2)v^{2} + \frac{1}{2}(\frac{0.01}{0.1^{2}})v^{2} = 25(3)$$

$$v = 5.5 \text{ m/s}$$



 $|^{y(m)}$ 

### Part II: Long Problems (5 points each)

**LP1.** Two small balls are connected by <u>a very light</u> rigid rod, as shown. The system rotates around the x - axis.

a) Find the moment of inertia of the system about the axis of rotation.

$$I = m_1 y_1^2 + m_2 y_2^2 = (0.3)(0.2)^2 + (0.2)(0.3)^2 = 0.03 \ kg \cdot m^2$$

$$0.2 \quad m_1 = 0.3 \ kg$$

$$-0.1 - \frac{0.1 - \frac{1}{0.2 - \frac{1}{$$

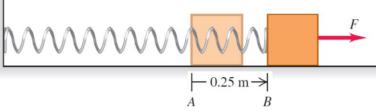
b) The system starts to rotate **from rest** with a constant angular acceleration of  $20 rad/s^2$ . Find the rotational kinetic energy of the system at t = 3 s.

$$\omega_f = \omega_i + \alpha t$$
  
= 0 + 20(3) = 60 rad/s  
$$K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.03)(60)^2 = 54 J$$

c) Find the tangential speed of  $m_1$  at t = 3 s.

$$v = R\omega = 0.2(60) = 12 m/s$$

**LP2.** A 2 kg block **rests** on **a frictionless horizontal** surface is attached to a spring of force constant k = 40 N/m, as shown. The spring is relaxed at point A, you move the block by pulling with a constant horizontal force F = 20 N.



(a) Find the total work done on the block as it moves from point A to point B.

$$\sum W = W_F + W_{F_S} = F(d) + \frac{1}{2}k(x_i^2 - x_f^2)$$
$$= 20(0.25) + \frac{1}{2}(40)(0_i^2 - 0.25^2) = 3.75 J$$

(b) Find the block's speed when it reaches point B.

$$\sum W = \Delta K$$
  
3.75 =  $\frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}(2)(v_f^2 - 0^2) \Rightarrow v_f = 1.9 m/s$ 

(c) If the surface was rough, then the <u>total</u> work done on the block as it moves from point A to point B will be

(\*) less than the value obtained in part a.

- \* greater than the value obtained in part a.
- \* equal to the value obtained in part a.

**LP3.** block 1  $(m_1 = 3 kg)$  moves on **a horizontal frictionless surface** with an initial velocity  $\vec{v}_{1_i} = (5\hat{\imath} - 4\hat{\jmath}) m/s$ . It collides with block 2  $(m_2 = 2 kg)$ , which **is initially at rest**. After the collision, the velocity of  $m_1$  is  $\vec{v}_{1_f} = (4\hat{\imath} - 2\hat{\jmath}) m/s$ .

(a) Find the velocity of block 2 after the collision in unit vector notation.

$$m_1 \vec{v}_{1_i} + m_2 \vec{v}_{2_i} = m_1 \vec{v}_{1_f} + m_2 \vec{v}_{2_f}$$
  

$$3(5\hat{\imath} - 4\hat{\jmath}) + 0 = 3(4\hat{\imath} - 2\hat{\jmath}) + 2\vec{v}_{2_f}$$
  

$$\vec{v}_{2_f} = \frac{(15\hat{\imath} - 12\hat{\jmath}) - (12\hat{\imath} - 6\hat{\jmath})}{2} = (1.5\hat{\imath} - 3\hat{\jmath}) m/s$$

(b) If the collision lasts for 0.1s, find the average net force exerted on block 1 during the collision in unit vector notation.

$$\left(\sum F\right)_{av_{on \,block\,1}} = \frac{\Delta \vec{p}_1}{\Delta t} = \frac{m_1\left(\vec{v}_{1_f} - \vec{v}_{1_i}\right)}{\Delta t} = \frac{3((4\hat{\iota} - 2\hat{j}) - (5\hat{\iota} - 4\hat{j}))}{0.1} = (-30i + 60j) N$$

(c) Find the change in kinetic energy of the two blocks system due to the collision.

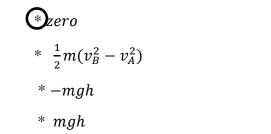
$$\begin{aligned} v_{1_i}^2 &= 5^2 + 4^2 = 41 \ (m/s)^2 \\ v_{1_f}^2 &= 4^2 + 2^2 = 20 \ (m/s)^2 \\ v_{2_f}^2 &= 1.5^2 + 3^2 = 11.25 \ (m/s)^2 \\ \Delta K &= (\frac{1}{2}m_1v_{1_f}^2 + \frac{1}{2}m_2v_{2_f}^2) - (\frac{1}{2}m_1v_{1_i}^2 + \frac{1}{2}m_2v_{2_i}^2) \\ &= \left(\frac{1}{2}(3)(20) + \frac{1}{2}(2)(11.25)\right) - \left(\frac{1}{2}(3)(41) + 0\right) = -20.25 J \end{aligned}$$

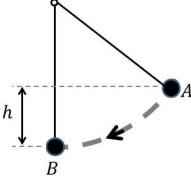
#### Part III: Questions (Choose the correct answer, one point each)

Q1. A box starts from rest at t = 0 s and moves along the positive x-axis with constant acceleration. If the speed of the box at t = 1 s is v, then its speed at t = 3 s is:

\* 
$$v$$
 \*  $2v$  \*  $3v$  \*  $9v$ 

Q2. When the bob (mass = m) of a simple pendulum moves from point A to point B, as shown, the work done on the bob by the **tension** is equal to





Q3. An object is **initially moving to the right** along +x-axis with velocity  $v_{xi}$ . A net force is exerted on the object giving it **an impulse towards the left**. After the impulse, which of following statements is always true about the final velocity of the object  $v_{xf}$ :

- $v_{xf}$  is negative.
- $v_{xf}$  is zero.
- $v_{xf}$  is positive.
- All the above are possible.

**Q4.** An object is moving in a circular path counterclockwise and **speeding up at a constant rate**. The object passes point A and then point B. Which figure represents the total acceleration of the object at these two points?

