



**Final Examination
Summer Semester 2022 – 2023**

July 24, 2023

Time: 5:00 – 7:00 PM

Name: Student No:

Section No: Serial No:

Instructors: Drs. Alaa Alfaiakawi, Afshin Hadipour, & Peter Lajko

Fundamental constants

$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2$	(Coulomb constant)
$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)$	(Permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$	(Permeability of free space)
$ e = 1.60 \times 10^{-19} \text{ C}$	(Elementary unit of charge)
$N_A = 6.02 \times 10^{23}$	(Avogadro's number)
$g = 9.8 \text{ m/s}^2$	(Acceleration due to gravity)
$m_e = 9.11 \times 10^{-31} \text{ kg}$	(Electron mass)
$m_p = 1.67 \times 10^{-27} \text{ kg}$	(Proton mass)

Prefixes of units

$m = 10^{-3}$	$\mu = 10^{-6}$	$n = 10^{-9}$	$p = 10^{-12}$
$k = 10^3$	$M = 10^6$	$G = 10^9$	$T = 10^{12}$

For use by Instructors only

Problems	1	2	3	4	5	6	7	8	9	10	Questions	Total
Marks												

Instructions to the Students:

1. Mobile or other electronic devices are **strictly prohibited** during the exam.
2. Programmable calculators, which can store equations, are not allowed.
3. Cheating incidents will be processed according to the university rules.

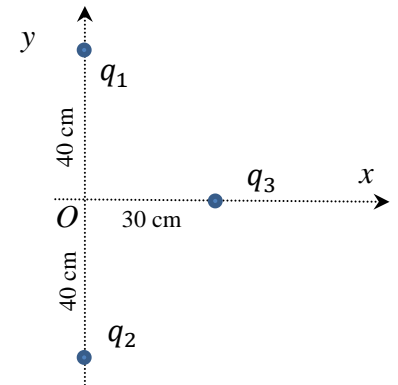
PART I: Solve the following problems. Show your solutions in detail.

1. Three point charges, $q_1 = q_2 = -8 \mu\text{C}$, and $q_3 = 8 \mu\text{C}$, are placed on the xy -plane, as shown. Calculate the x and y components of the net electric force, \vec{F}_3 , acting on q_3 . **[4 points]**

$$r_{13} = r_{23} = \sqrt{(0.3\text{m})^2 + (0.4\text{m})^2} = 0.5 \text{ m}$$

$$F_{3,y} = F_{13,y} + F_{23,y} = k \frac{|q_1 q_3|}{r_{12}^2} \sin(\theta) - k \frac{|q_2 q_3|}{r_{23}^2} \sin(\theta) = 0$$

$$F_{3,x} = F_{13,x} + F_{23,x} = -2k \frac{|q_1 q_3|}{r_{13}^2} \cos(\theta) = -2k \frac{|q_1 q_3|}{r_{13}^2} \frac{3}{5} = -2.8 \text{ N}$$

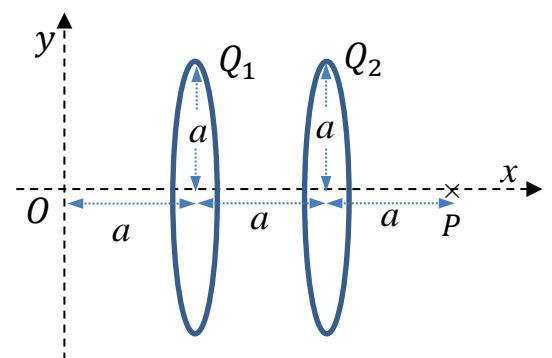


2. Two uniformly charged rings of identical radii a and of charges $Q_1 = -4 \text{ nC}$ and $Q_2 = 4 \text{ nC}$ are placed along the x -axis as shown. If $a = 0.4 \text{ m}$, calculate the net electric field vector at point P . **[3 points]**

$$\vec{E}_1 = \frac{kQ_1 2a}{((2a)^2 + a^2)^{3/2}} \hat{i} = \left(-40.25 \frac{\text{N}}{\text{C}}\right) \hat{i}$$

$$\vec{E}_2 = \frac{kQ_2 a}{(a^2 + a^2)^{3/2}} \hat{i} = \left(79.55 \frac{\text{N}}{\text{C}}\right) \hat{i}$$

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 = \left(39.3 \frac{\text{N}}{\text{C}}\right) \hat{i}$$



3. A spherical shell of inner radius $a = 5$ cm and outer radius $b = 10$ cm has uniform volume charge density $\rho = -90$ nC/m³. A point charge q is placed to the center of the shell. At a radial distance $r = 20$ cm from the center, the net electric field points away from the center and has a magnitude of $E = 240$ N/C. Determine the value of q . [3 points]

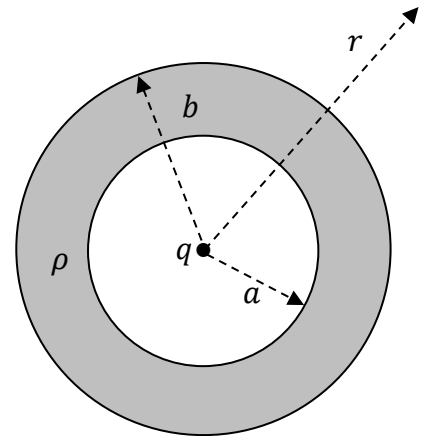
Gauss's Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q_{encl}}{\epsilon_0}$$

$$Q_{encl} = q + \rho \left(\frac{4}{3}\pi(b^3 - a^3) \right)$$

$$q = E(4\pi r^2)\epsilon_0 - \rho \left(\frac{4}{3}\pi(b^3 - a^3) \right) \Rightarrow q = 1.4 \text{ nC}$$



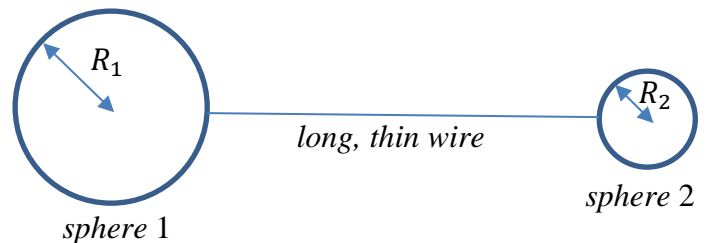
4. Two conducting spheres of radii $R_1 = 8$ cm and $R_2 = 4$ cm are placed far from each other. Initially, the *sphere 1* has charge $Q_0 = 60$ μ C and *sphere 2* is uncharged. Then the spheres are connected to each other with a long, thin wire. Calculate the final charge of the sphere of radius R_2 . [4 points]

$$V_1 = V_2 \Rightarrow \frac{kQ_1}{R_1} = \frac{kQ_2}{R_2}$$

$$Q_0 = Q_1 + Q_2 \text{ so}$$

$$\frac{Q_1}{Q_2} = \frac{R_1}{R_2} \Rightarrow \frac{Q_1}{Q_0 - Q_1} = \frac{R_1}{R_2} \Rightarrow$$

$$Q_2 = 20 \mu\text{C}$$

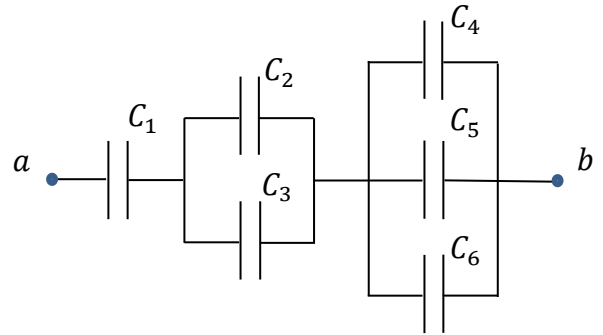


5. Calculate the equivalent capacitance between points a and b of the following network of six identical capacitors, $C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = 4 \mu\text{F}$. **[3 points]**

$$C_{23} = C_2 + C_3 = 8 \mu\text{F}$$

$$C_{456} = C_4 + C_5 + C_6 = 12 \mu\text{F}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_{456}} \Rightarrow C_{eq} = 2.2 \mu\text{F}$$



6. A cylindrical conducting wire of length $L = 5.0 \text{ m}$ and radius $r = 0.1 \text{ mm}$ has resistance $R = 4 \Omega$. The wire has a concentration of free electrons $n = 1.5 \times 10^{28} \text{ m}^{-3}$. If the magnitude of electric field in the wire is $E = 0.05 \text{ N/C}$, calculate the drift speed v_d of the electrons. **[4 points]**

$$R = \rho \frac{L}{A} = \rho \frac{L}{\pi r^2} \Rightarrow \rho = \frac{R\pi r^2}{L}$$

$$\rho = 2.5 \times 10^{-8} \Omega \text{ m}$$

$$E = \rho J \Rightarrow J = \frac{E}{\rho} = 2 \times 10^6 \text{ A/m}^2$$

$$J = nev_d \Rightarrow v_d = \frac{J}{ne} = 8.3 \times 10^{-4} \text{ m/s}$$

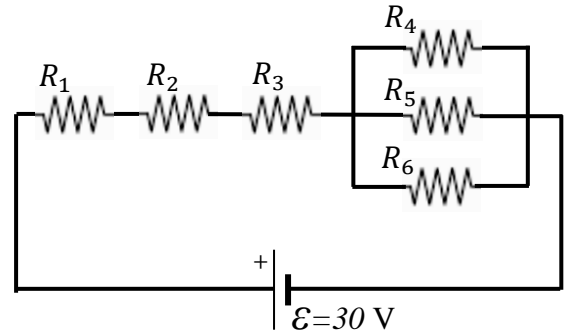
7. Six identical resistors, $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 3 \Omega$, are connected into circuit as shown. Find the power supplied by the ideal *emf* device. [4 points]

$$R_{123} = R_1 + R_2 + R_3 = 9 \Omega$$

$$R_{456} = \frac{1}{\frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}} = 1 \Omega$$

$$R_{eq} = R_{123} + R_{456} = 10 \Omega$$

$$P_{sup} = \frac{\mathcal{E}^2}{R_{eq}} = 90 \text{ W}$$

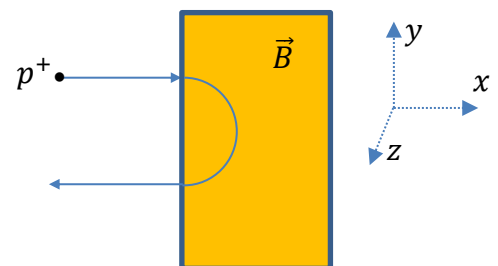


8. A proton moves into a region of uniform magnetic field with a velocity of $\vec{v} = \left(1500 \frac{\text{m}}{\text{s}}\right) \hat{i}$ and leaves it by moving to opposite direction, as shown. The magnetic field is perpendicular to the xy -plane. If the proton travels a distance $s = 0.8 \text{ m}$ in the magnetic field, find the magnitude and direction of the uniform magnetic field. [3 points]

$s = \pi R$, since the proton completes a semicircle.

$$R = \frac{s}{\pi} = \frac{mv}{|q|B} \Rightarrow B = \frac{mv}{|q|R} = 61.5 \mu\text{T}$$

\vec{B} is outward (or $\vec{B} = B_z \hat{k}$).

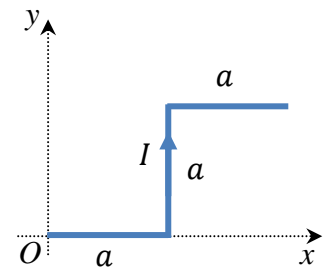


9. A three-segment wire of total length $3a$ carries a current $I = 3 \text{ A}$, as shown in the figure. The wire is in uniform magnetic field, $\vec{B} = (2 \text{ T})\hat{k}$. If $a = 0.4 \text{ m}$, calculate the net magnetic force acting on the wire.

[3 points]

$$\vec{L}_{eff} = 2a\hat{i} + a\hat{j}$$

$$\vec{F}_B = I\vec{L}_{eff} \times \vec{B} = (2.4 \text{ N})\hat{i} + (-4.8 \text{ N})\hat{j}$$



10. Three very long, parallel wires are perpendicular to the xy -plane. Each wire carries a current of magnitude $I = 4 \text{ A}$ in the direction as shown. Calculate the force per unit length vector $\frac{\vec{F}_3}{L}$ acting on the wire of I_3 .

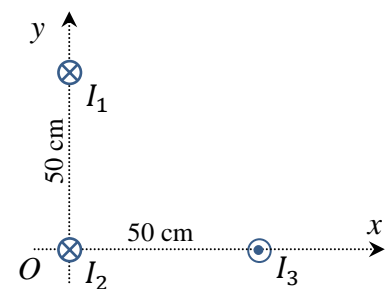
[5 points]

$$\frac{F_{13,x}}{L} = \frac{\mu_0 I_1 I_3}{2\pi 0.5\text{m}\sqrt{2}} \cos(-45^\circ) = 3.2 \times 10^{-6} \text{ N/m}$$

$$\frac{F_{13,y}}{L} = \frac{\mu_0 I_1 I_3}{2\pi 0.5\text{m}\sqrt{2}} \sin(-45^\circ) = -3.2 \times 10^{-6} \text{ N/m}$$

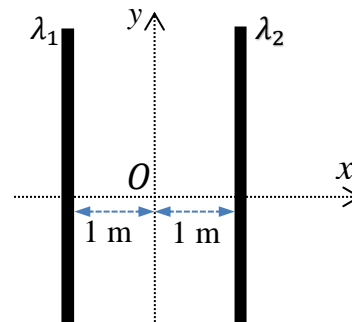
$$\frac{F_{23,x}}{L} = \frac{\mu_0 I_2 I_3}{2\pi 0.5\text{m}} = 6.4 \times 10^{-6} \text{ N/m}, \quad \frac{F_{23,y}}{L} = 0$$


$$\frac{\vec{F}_3}{L} = \left(9.6 \times 10^{-6} \frac{\text{N}}{\text{m}}\right)\hat{i} + \left(-3.2 \times 10^{-6} \frac{\text{N}}{\text{m}}\right)\hat{j}$$



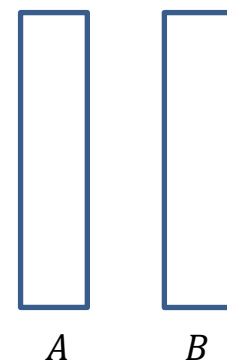
PART II: Conceptual Questions (each carries 1 point). Tick the best answer:


1. Two very long uniformly charged lines with linear charge densities $\lambda_1 = 9 \text{ nC/m}$ and $\lambda_2 = -6 \text{ nC/m}$ are placed perpendicular to the x -axis, as shown. Along the x -axis, the net electric field vector, \vec{E}_{net} , is zero



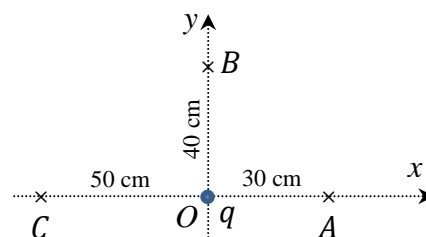
- a) somewhere to the left of λ_1 .
 b) somewhere to the right of λ_2 . 
 c) somewhere between the two linear charge densities.
 d) everywhere.


2. Two very large parallel charged conducting plates A and B have identical charges Q and Q . Which statement is correct for the location of the charges? The charges are



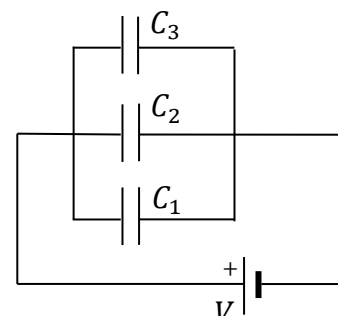
- a) on the right surface of plate A and on the right surface of plate B .
 b) on the right surface of plate A and on the left surface of plate B .
 c) on the left surface of plate A and on the right surface of plate B . 
 d) on the left surface of plate A and on the left surface of plate B .


3. A point charge $q > 0$ is fixed at the origin as shown. Which relation is correct for the electric potential differences?



- a) $V_A - V_C < V_A - V_B$.
 b) $V_B - V_C < V_A - V_C$. 
 c) $V_A - V_C < V_B - V_C$.
 d) $V_A - V_B < V_C - V_B$.

4. Three capacitors are connected into a circuit as shown. If $C_1 = C$, $C_2 = 2C$, $C_3 = 3C$, which relation is correct for the charges stored on the capacitors?



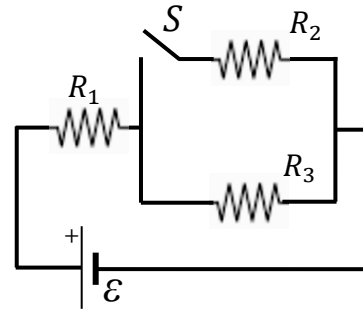
- a) $Q_1 = Q_2$.
 b) $Q_1 = Q_3$.
 c) $Q_1 = Q_2/2$. 
 d) $Q_1 = Q_3/2$.

5. A cylindrical wire has length L , radius r and resistivity ρ . Another cylindrical wire made of the same material has length $2L$, radius $2r$, so, its resistivity is

- a) ρ . ←
- b) 2ρ .
- c) 4ρ .
- d) $\rho/2$.

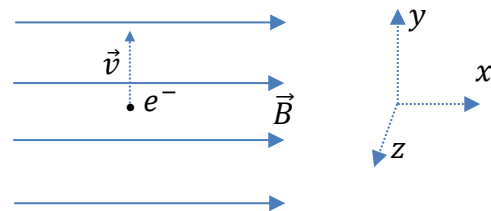
6. In the circuit shown, $R_1 = R_2 = R_3$. If the switch is closed, the potential change on R_1

- a) will decrease.
- b) will increase. ←
- c) will remain the
- d) will drop to zero.



7. An electron moves in a region of uniform magnetic field, $\vec{B} = B_x \hat{i}$. The velocity of the electron is momentarily parallel with the positive y -axis, as shown. At this moment, the magnetic force \vec{F}_B acting on the electron is along the

- a) negative x -axis.
- b) negative y -axis.
- c) negative z -axis.
- d) positive z -axis. ←



8. A current I flows in a very long straight wire along the negative y -axis, as shown. Due to this current, the magnetic field \vec{B} points parallel with the negative z -axis

- a) at point A . ←
- b) at point B .
- c) at both points A and B .
- d) at none of the points A and B .

