


**Second Midterm Examination**  
**Spring Semester 2023 – 2024**
**May 4, 2024**
**Time: 12:00 – 1:30 PM**

Name: ..... Student No: .....

Section No: ..... Serial No: .....

Instructors: Drs. Alfailakawi, Lajko, Sharma, Vagenas

**Fundamental constants**

$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2$	(Coulomb constant)
$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)$	(Permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m}/\text{A}$	(Permeability of free space)
$ e  = 1.60 \times 10^{-19} \text{ C}$	(Elementary unit of charge)
$N_A = 6.02 \times 10^{23}$	(Avogadro's number)
$g = 9.8 \text{ m/s}^2$	(Acceleration due to gravity)
$m_e = 9.11 \times 10^{-31} \text{ kg}$	(Electron mass)
$m_p = 1.67 \times 10^{-27} \text{ kg}$	(Proton mass)

**Prefixes of units**

$m = 10^{-3}$	$\mu = 10^{-6}$	$n = 10^{-9}$	$p = 10^{-12}$
$k = 10^3$	$M = 10^6$	$G = 10^9$	$T = 10^{12}$

**For use by Instructors only**

Problems	1	2	3	4	5	6	7	8	Questions	Total
Marks										

**Instructions to the Students:**

1. Mobile or other electronic devices are **strictly prohibited** during the exam.
2. Programmable calculators, which can store equations, are not allowed.
3. **Cheating incidents will be processed according to the university rules.**

**PART I: Solve the following problems. Show your solutions in detail.**

1. Three point charges  $q_1 = 4 \text{ nC}$ ,  $q_2 = 5 \text{ nC}$ , and  $q_3 = 4 \text{ nC}$  are placed on the  $x$  and  $y$ -axis as shown. Compute the work done by the external agent to move the electric charge  $q = 3 \text{ nC}$  from infinity ( $V_\infty = 0 \text{ V}$ ) to point  $A$ , without changing its kinetic energy. **[4 points]**

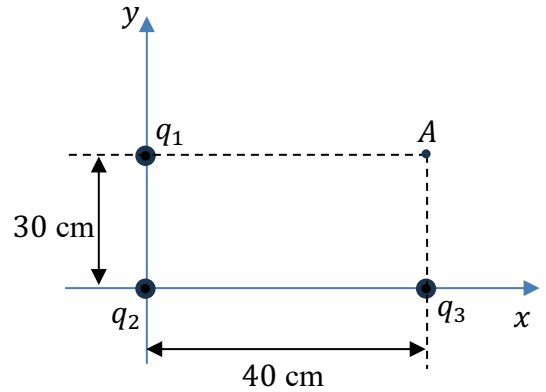
$$d = \sqrt{(0.40)^2 + (0.30)^2} \text{ m} \Rightarrow d = 0.5 \text{ m}$$

$$W_{\infty \rightarrow A}^{ext} = -W_{\infty \rightarrow A}^{el.field} = -q(V_\infty - V_A) = qV_A \quad [1]$$

$$V_A = k \frac{q_1}{0.40} + k \frac{q_2}{0.50} + k \frac{q_3}{0.30} \quad [1]$$

$$V_A = 300 \text{ V} \quad [1]$$

$$W_{\infty \rightarrow A}^{ext} = qV_A \Rightarrow W_{\infty \rightarrow A}^{ext} = 0.9 \text{ } \mu\text{J} \quad [1]$$



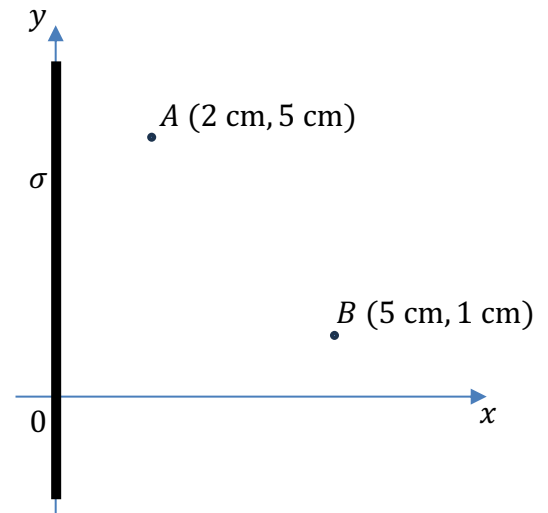
2. An infinite uniformly charged plane with surface charge density  $\sigma = 35.4 \text{ nC/m}^2$  is placed perpendicular to the  $x$ -axis as shown. Compute the potential difference  $V_A - V_B$ . **[4 points]**

$$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{l} \quad [1]$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{i} \quad d\vec{l} = dx\hat{i} + dy\hat{j} \quad [1]$$

$$V_A - V_B = \int_A^B \left[ \frac{\sigma}{2\epsilon_0} \hat{i} \right] \cdot (dx\hat{i} + dy\hat{j}) = \frac{\sigma}{2\epsilon_0} \int_{2\text{cm}}^{5\text{cm}} dx \quad [1]$$

$$V_A - V_B = \frac{\sigma}{2\epsilon_0} (0.05 - 0.02) \Rightarrow V_A - V_B = 60 \text{ V} \quad [1]$$



3. In the given network of capacitors with  $C_1 = C_2 = C_3 = 10 \mu\text{F}$ , and  $C_4 = C_5 = 5 \mu\text{F}$ , the capacitor  $C_2$  has charge  $Q_2 = 15 \mu\text{C}$ . Find the stored energy in capacitor  $C_5$ . **[4 points]**

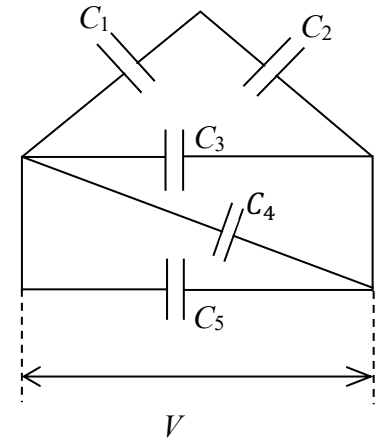
$C_1$  and  $C_2$  are in series:

$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = 5 \mu\text{F} \quad [1]$$

so, the total charge of  $C_{12}$  is  $Q_{12} = Q_1 = Q_2$ . [1]

$$C_{12} = \frac{Q_{12}}{V} \Rightarrow V = \frac{Q_{12}}{C_{12}} \Rightarrow V = 3 \text{ V} \quad [1]$$

$$U_{C_5} = \frac{1}{2} C_5 V^2 \Rightarrow U_{C_5} = 22.5 \mu\text{J} \quad [1]$$



4. An air-filled parallel-plate capacitor with a plate area  $A$  and separation  $d = 5 \text{ cm}$ , has capacitance  $C_0 = 40 \mu\text{F}$ . Then, the capacitor is partially filled with a dielectric with constant  $K = 3$  as shown. Find the equivalent capacitance. **[4 points]**

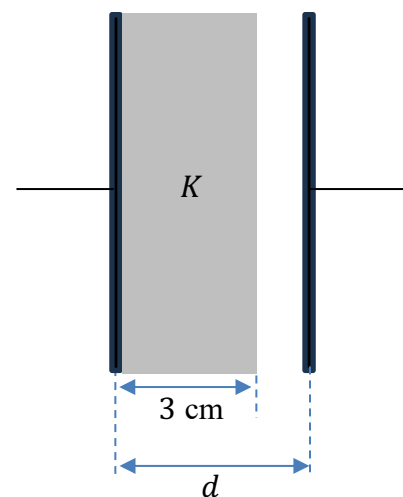
$$C_0 = \epsilon_0 \frac{A}{d} \Rightarrow \epsilon_0 A = C_0 d \quad [1]$$

$$C_1 = K \epsilon_0 \frac{A}{x} \Rightarrow C_1 = K \frac{C_0 d}{0.03} \Rightarrow C_1 = 200 \mu\text{F} \quad [1]$$

$$C_2 = \epsilon_0 \frac{A}{d-x} \Rightarrow C_2 = \frac{C_0 d}{0.05-0.03} \Rightarrow C_2 = 100 \mu\text{F} \quad [1]$$

$C_1$  and  $C_2$  are in series:

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \Rightarrow C_{eq} = 66.7 \mu\text{F} \quad [1]$$



5. In a metallic wire, the electric current is  $I(t) = 12t + 3$  ( $t$  is in seconds and  $I$  is in amperes). Find the number of electrons that flow through a cross-sectional area of the metallic wire between the time  $t_1 = 1$  ns and  $t_2 = 3$  ns. [4 points]

$$I = \frac{dQ}{dt} \Rightarrow Idt = dQ \Rightarrow Q = \int_{1\text{ ns}}^{3\text{ ns}} I(t)dt \quad [1]$$

$$Q = \int_{1\text{ ns}}^{3\text{ ns}} (12t + 3)dt \Rightarrow Q = 12 \left[ \frac{t^2}{2} \right]_{1\text{ ns}}^{3\text{ ns}} + 3[t]_{1\text{ ns}}^{3\text{ ns}} \quad [1]$$

$$Q = 6 \text{ nC} \quad [1]$$

**Electric charge is quantized:**

$$Q = N |e| \Rightarrow N = \frac{Q}{|e|} \Rightarrow N = 3.75 \times 10^{10} \text{ electrons} \quad [1]$$

6. Find the electric currents in all branches of the given electric circuit. [5 points]

**Junction rule:**

$$I_3 = I_1 + I_2 \quad [1]$$

**Left loop:**

$$20 - 6I_1 - 5I_3 - 9I_1 = 0$$

Substituting the first equation in the second one:

$$20 - 20I_1 - 5I_2 = 0 \Rightarrow 4I_1 + I_2 = 4 \quad [1]$$

**Right loop:**

$$15 - 10I_2 - 5I_3 = 0$$

Substituting the first equation in the above:

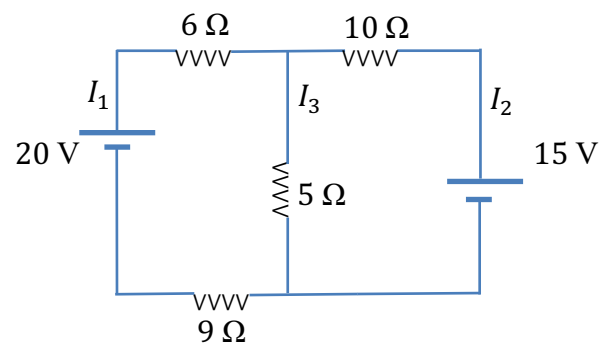
$$15 - 15I_2 - 5I_1 = 0 \Rightarrow I_1 + 3I_2 = 3 \quad [1]$$

Multiplying the 3<sup>rd</sup> equation with (-3) and adding it to the above equation, one gets:

$$-11I_1 = -9 \Rightarrow I_1 = 0.82 \text{ A} \quad [1]$$

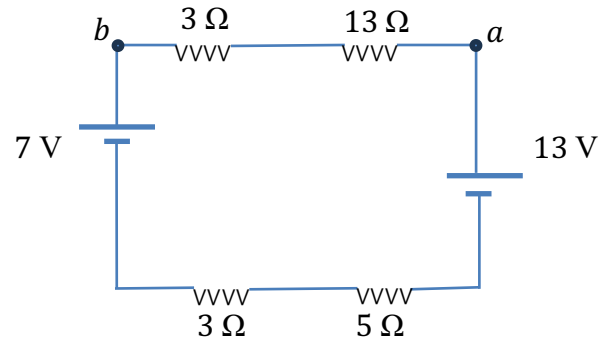
Substituting  $I_1$  in the 2<sup>nd</sup> equation, one gets:

$$4 \times 0.82 + I_2 = 4 \Rightarrow I_2 = 0.73 \text{ A} \text{ and so from the first equation: } I_3 = 1.55 \text{ A} \quad [1]$$



7. Find the power dissipated between  $a$  and  $b$ .

[3 points]



**Loop rule:**

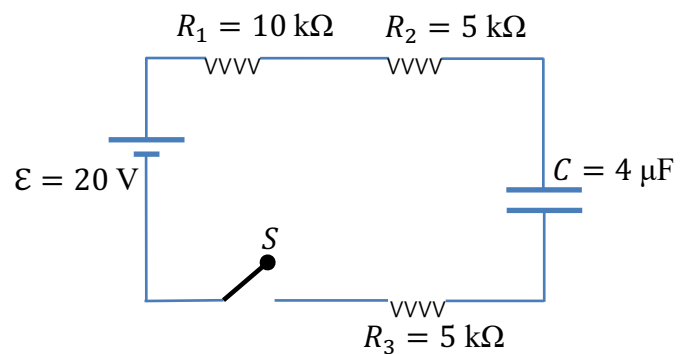
$$13 - 13I - 3I - 7 - 3I - 5I = 0 \Rightarrow 24I = 6 \Rightarrow I = 0.25 \text{ A} \quad [1]$$

$$V_a - 13I - 3I = V_b \Rightarrow V_a - V_b = 4 \text{ V} \Rightarrow V_{ab} = 4 \text{ V} \quad [1]$$

$$P = V_{ab}I \Rightarrow P = 1.0 \text{ W} \quad [1]$$

8. At  $t = 0$  s, the switch  $S$  is closed. Find the electric charge stored in the capacitor at  $t = 0.02$  s.

[4 Points]



After infinite time, the capacitor is fully charged:

$$Q = CV_C \Rightarrow Q = 80 \mu\text{C} \quad [1]$$

The capacitor will be charged through the three resistances.

$$R_{eq} = R_1 + R_2 + R_3 = 20 \text{ k}\Omega \quad [1]$$

So, the time constant is:  $\tau = R_{eq}C \Rightarrow \tau = 0.08 \text{ s}$

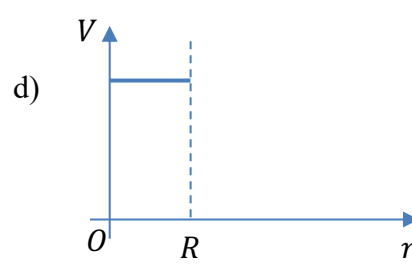
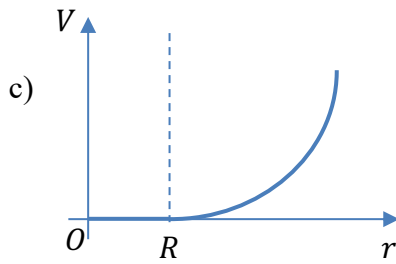
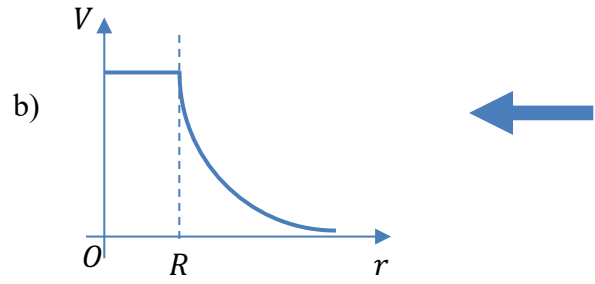
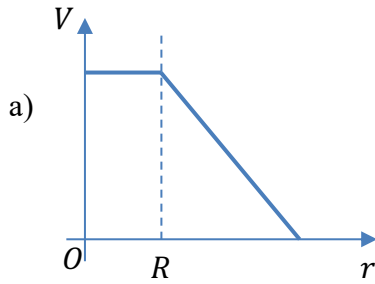
The electric charge stored in the capacitor at  $t = 0.02$  s is:

$$q(t) = Q[1 - e^{-\frac{t}{\tau}}] \Rightarrow q(t) = 80[1 - e^{-\frac{0.02}{0.08}}] \mu\text{C} \quad [1]$$


$$q(t) = 17.7 \mu\text{C} \quad [1]$$

**PART II: Conceptual Questions (each carries 1 point). Tick the best answer:**


1. Which is the diagram that shows the electric potential of a charged conducting sphere of radius  $R$  as a function of distance  $r$  from the center of the conducting sphere?




2. An infinite conducting plane is uniformly charged. Its equipotential surfaces are:

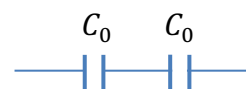
- a) Infinite inclined planes making an angle of  $45^\circ$  with the conducting plane.
- b) infinite inclined planes making an angle of  $60^\circ$  with the conducting plane.
- c) Infinite planes parallel to the conducting plane. 
- d) Infinite planes perpendicular to the conducting plane.

3. An air-filled parallel-plate capacitor is charged by a battery. While the battery remains connected, the separation between the plates of the capacitor is increased. Which statement is correct?

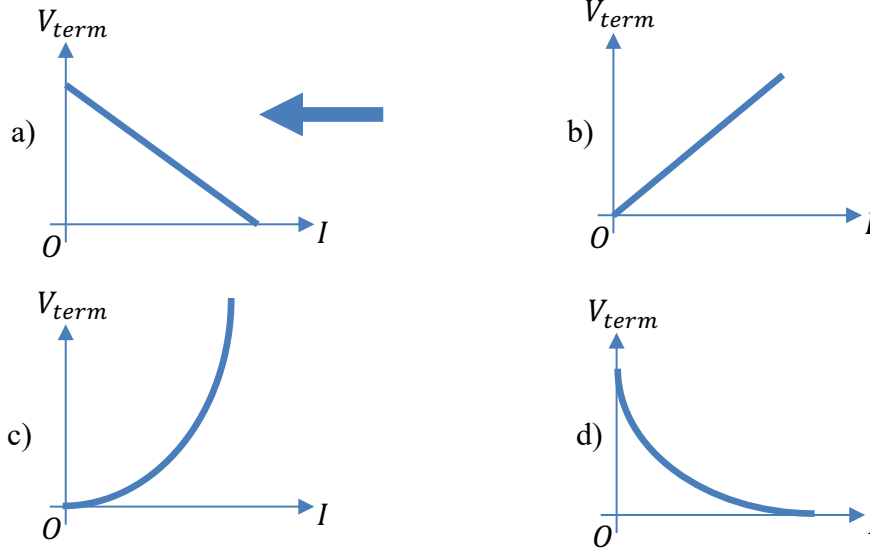
- a) The energy density of the capacitor will remain the same.
- b) The electric charge of the capacitor will increase.
- c) The electric energy of the capacitor will increase.
- d) The energy density of the capacitor will decrease. 

4. Two parallel-plate capacitors are connected as shown. Then, a dielectric is inserted in one of the capacitors and fully fills it. The equivalent capacitance of the two capacitors will

- a) increase. 
- b) decrease.
- c) remain the same.
- d) become zero.



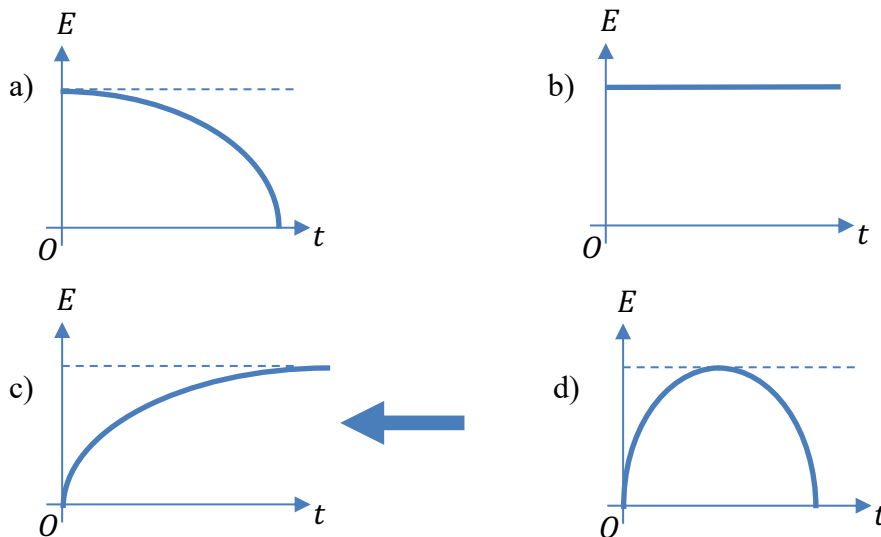
5. Which diagram gives the terminal voltage of a real source of emf as a function of the electric current?



6. The power input to a real source of emf is:

- a)  $P_{in} = \mathcal{E} - Ir.$
- b)  $P_{in} = \mathcal{E}I - I^2r.$
- c)  $P_{in} = \mathcal{E}I + I^2r.$  ←
- d)  $P_{in} = \mathcal{E}I + I^2r^2.$

7. In an RC circuit, during the charging of the capacitor, the diagram of the magnitude of the electric field between the plates of the capacitor as a function of the time is:



8. In a charging RC circuit, the time constant is  $\tau$ . If the switch is closed at  $t = 0$ , when the capacitor is uncharged, after how much time will the charge be the 63.2 % of its maximum value  $Q$ ?

- a)  $\tau.$  ←
- b)  $2\tau.$
- c)  $5\tau.$
- d)  $10\tau.$