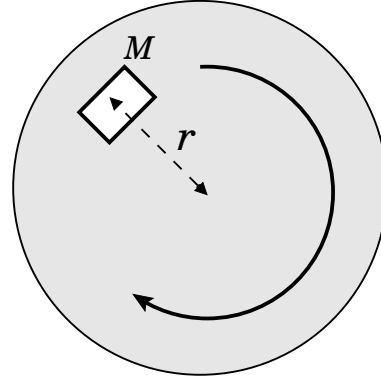


1. A box of mass $M = 0.1$ kg rests at a distance of $r = 25$ cm from the centre of a horizontal rotating table. The box remains at its place until the frequency of rotation reaches 45 rpm. Find the coefficient of static friction (μ_s) between the box and the table. 4 points



Solution: We have

$$f = 45/60 = 0.75 \text{ s}^{-1}$$

$$v = 2\pi r f = 1.18 \text{ m/s}$$

The force equation,

$$\mu_s M g = \frac{M v^2}{r} \implies \mu_s = \frac{v^2}{r g} = 0.57$$

2. A bicycle tire with radius $r = 50$ cm makes 64 revolutions while its speed uniformly reduces from 36 km/h to complete stop. Find the angular acceleration of the tire. 4 points

Solution: The initial speed and the initial angular speed are

$$v_0 = 36/3.6 = 10 \text{ m/s}$$

$$\omega_0 = \frac{v_0}{r} = 20 \text{ rad/s}$$

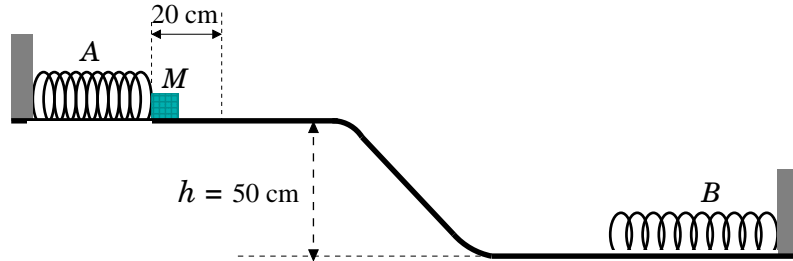
The total angle covered is

$$\theta - \theta_0 = N \times 2\pi = 402 \text{ rad}$$

Then

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \implies \alpha = \frac{\omega^2 - \omega_0^2}{2(\theta - \theta_0)} = -0.5 \text{ rad/s}^2$$

3. A 2-kg box is used to compress the spring A with stiffness constant $k_A = 850 \text{ N/m}$ by 20 cm. When the box is released it slides down a frictionless path and hits spring B with stiffness constant $k_B = 600 \text{ N/m}$. Find the compression in spring B when the box comes momentarily to rest. 4 points



Solution: The work-energy principle is

$$\begin{aligned}
 \text{KE}_i + \text{PE}_i &= \text{KE}_f + \text{PE}_f \\
 \implies 0 + Mgh + \frac{1}{2}k_A x_A^2 &= 0 + 0 + \frac{1}{2}k_B x_B^2 \\
 \implies 2 \times 9.8 \times 0.5 + \frac{1}{2} \times 850 \times (0.2)^2 &= \frac{1}{2} \times 600 \times x_B^2 \\
 \implies 26.8 &= \frac{1}{2} \times 600 \times x_B^2 \\
 \implies x_B &= 0.30 \text{ m}
 \end{aligned}$$

4. A 1000-kg car moving on a horizontal road accelerates uniformly from rest to a speed of 15 m/s in 8 s during which it travels a distance of 60 m. The average power of the car's engine is 20 kW. Find the average force of friction on the car. 4 points

Solution: We have

$$P_{\text{engine}} = \frac{W_{\text{engine}}}{t} \implies W_{\text{engine}} = P_{\text{engine}} t = (20 \times 10^3) \times 8 = 1.6 \times 10^5 \text{ J}$$

The work-energy principle gives

$$W_{\text{engine}} + W_{fr} = \frac{1}{2} M v^2 \implies W_{fr} = \frac{1}{2} M v^2 - W_{\text{engine}} = -4.75 \times 10^4 \text{ J}$$

Work done by the force of friction is

$$W_{fr} = -F_{fr} d \implies F_{fr} = \frac{-W_{fr}}{d} = 792 \text{ N}$$

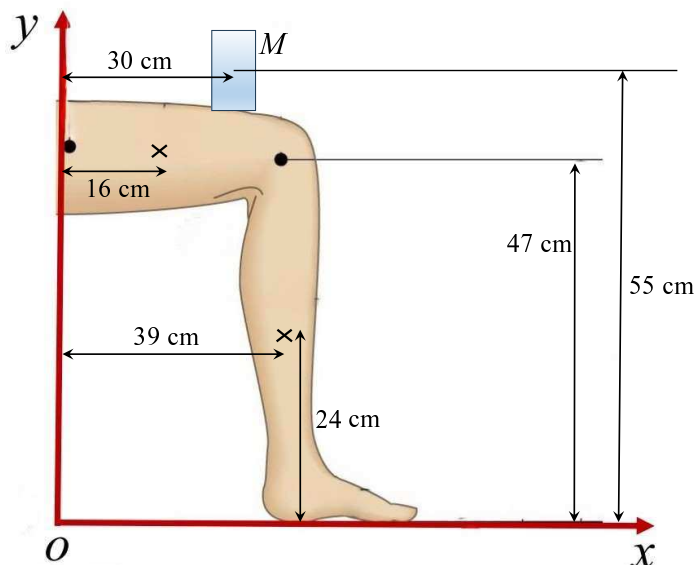
5. A pile of books of mass $M = 8$ kg is placed on a leg bent at 90° as shown. The masses of the upper leg, the lower leg (with the foot) are respectively $M_1 = 9$ kg and $M_2 = 7$ kg. The lengths of different parts are shown in the figure. Find the centre of mass of the leg holding the books. 4 points

Solution:

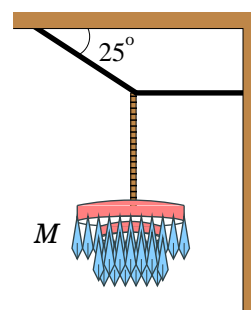
$M_1 = 9$ kg	$x_1 = 16$ cm	$y_1 = 47$ cm
$M_2 = 7$ kg	$x_2 = 39$ cm	$y_2 = 24$ cm
$M = 8$ kg	$x = 30$ cm	$y = 55$ cm

$$x_{CM} = \frac{M_1 x_1 + M_2 x_2 + M x}{M_1 + M_2 + M} = 27.4 \text{ cm}$$

$$y_{CM} = \frac{M_1 y_1 + M_2 y_2 + M y}{M_1 + M_2 + M} = 43.0 \text{ cm}$$



6. Two ropes support a hanging chandelier as shown. If the tension in the horizontal rope is 50 N, find the mass of the chandelier. 4 points

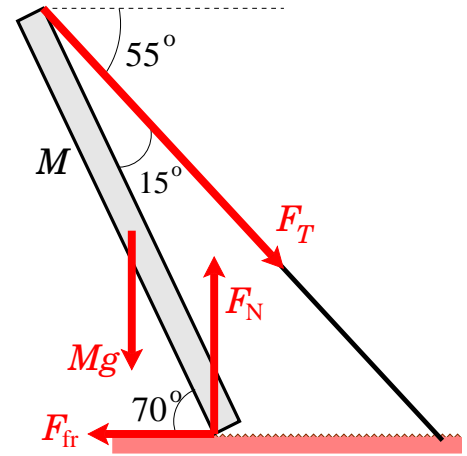


Solution: Let the tensions in the two ropes be F_{T1} (horizontal rope) and F_{T2} . Then

$$F_{T1} - F_{T2} \cos 25^\circ = 0 \implies F_{T2} = \frac{F_{T1}}{\cos 25^\circ} = 55 \text{ N}$$

$$F_{T2} \sin 25^\circ - Mg = 0 \implies M = \frac{F_{T2} \sin 25^\circ}{g} = 2.4 \text{ kg}$$

7. A 80-kg uniform bar is made to stand tilted at 70° to the horizontal on a rough horizontal surface by a cord connecting the top of the bar to the ground as shown. The system is in equilibrium. The forces acting on the bar are shown in the figure.



- (a) Find the tension in the cord. 2 points
- (b) Find the normal force on the bar by the ground. 2 points
- (c) Find the force of friction on the bar by the rough surface. 1 point

Solution: Let L be the length of the bar. We choose the pivot at the point where the bar touches the ground. Then the second condition of equilibrium gives us

$$+Mg \times \frac{L}{2} \times \sin 20^\circ - F_T \times L \times \sin 15^\circ = 0$$

$$\implies F_T = \frac{Mg \sin 20^\circ}{2 \sin 15^\circ} = 518 \text{ N}$$

Force (y - component): $-F_T \sin 55^\circ - Mg + F_N = 0$

$$\implies F_N = F_T \sin 55^\circ + Mg = 1208 \text{ N}$$

Force (x - component): $+F_T \cos 55^\circ - F_{fr} = 0 \implies F_{fr} = F_T \cos 55^\circ = 297 \text{ N}$