


**Second Midterm Examination
 Spring Semester 2022 – 2023**
April 29, 2023
Time: 12:00 – 1:30 PM

Name: Student No:

Section No: Serial No:

Instructors: Drs. Alfrousheh, Alfailakawi, Hadipour, Lajko, Sharma, Vagenas

Fundamental constants

$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2$	(Coulomb constant)
$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)$	(Permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A}$	(Permeability of free space)
$ e = 1.60 \times 10^{-19} \text{ C}$	(Elementary unit of charge)
$N_A = 6.02 \times 10^{23}$	(Avogadro's number)
$g = 9.8 \text{ m/s}^2$	(Acceleration due to gravity)
$m_e = 9.11 \times 10^{-31} \text{ kg}$	(Electron mass)
$m_p = 1.67 \times 10^{-27} \text{ kg}$	(Proton mass)

Prefixes of units

$m = 10^{-3}$	$\mu = 10^{-6}$	$n = 10^{-9}$	$p = 10^{-12}$
$k = 10^3$	$M = 10^6$	$G = 10^9$	$T = 10^{12}$

For use by Instructors only

Problems	1	2	3	4	5	6	7	8	Questions	Total
Marks										

Instructions to the Students:

1. Mobile or other electronic devices are **strictly prohibited** during the exam.
2. Programmable calculators, which can store equations, are not allowed.
3. **Cheating incidents will be processed according to the university rules.**

PART I: Solve the following problems. Show your solutions in detail.

1. An air-filled parallel plate capacitor C with a plate area 0.030 m^2 and a plate separation 1.0 mm is charged to a potential difference $V_0 = 20 \text{ V}$. The battery is then disconnected. A dielectric slab with $K = 3.2$ is then inserted between the plates filling the space completely. What is the energy stored on the capacitor now? **[4 points]**

$$C_0 = \epsilon_0 \frac{A}{d} = \frac{8.85 \times 10^{-12} \times 0.03}{1.0 \times 10^{-3}} \text{ F} = 2.66 \times 10^{-10} \text{ F}$$

$$Q_0 = C_0 V_0 = 2.66 \times 10^{-10} \text{ F} \times 20 \text{ V} = 5.31 \times 10^{-9} \text{ C}$$

$$C = K C_0 = 8.51 \times 10^{-10} \text{ F}$$

$$U = \frac{Q_0^2}{2C} = \frac{(5.31 \times 10^{-9} \text{ C})^2}{2 \times 8.51 \times 10^{-10} \text{ F}} = 1.66 \times 10^{-8} \text{ J}$$

2. If $C_1 = 12 \mu\text{F}$, $C_2 = 8 \mu\text{F}$, $C_3 = 16 \mu\text{F}$ and $V = 15 \text{ V}$, what is the charge stored on the capacitor C_3 ? **[5 points]**

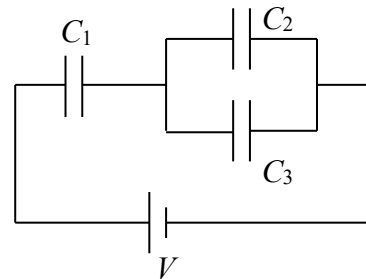
$$C_{23} = C_2 + C_3 = 8 \mu\text{F} + 16 \mu\text{F} = 24 \mu\text{F}$$

$$C_{eq} = \frac{C_1 \cdot C_{23}}{C_1 + C_{23}} = \frac{12 \mu\text{F} \times 24 \mu\text{F}}{(12 + 24) \mu\text{F}} = 8 \mu\text{F}$$

$$Q = C_{eq} V = 8 \mu\text{F} \times 15 \text{ V} = 120 \mu\text{C}$$

$$V_{C_2} = V_{C_3} = \frac{Q}{C_{23}} = \frac{120 \mu\text{C}}{24 \mu\text{F}} = 5 \text{ V}$$

$$Q_{C_3} = C_3 V_{C_3} = 16 \mu\text{F} \times 5 \text{ V} = 80 \mu\text{C}$$



3. An air-filled parallel-plate capacitor with a surface area A and a plate separation d , has capacitance C_0 . When the capacitor is filled with two dielectric materials with dielectric constants $K_1 = 2$ and $K_2 = 4$, as shown, the capacitance is 120 nF. Calculate the initial capacitance C_0 . [5 points]

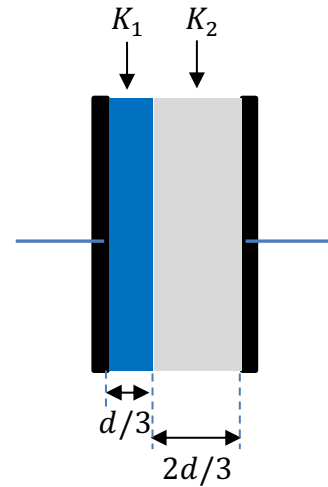
$$C_0 = \epsilon_0 \frac{A}{d}$$

$$C_1 = K_1 \epsilon_0 \frac{A}{d/3} = 3K_1 \epsilon_0 \frac{A}{d} = 6C_0$$

$$C_2 = K_2 \epsilon_0 \frac{A}{2d/3} = \frac{3}{2} K_2 \epsilon_0 \frac{A}{d} = 6C_0$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{eq} = 3C_0 \Rightarrow C_0 = 40 \text{ nF}$$



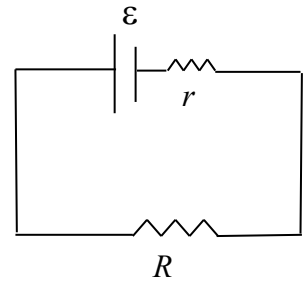
4. An electric field $E = 6.0 \text{ V/m}$ is applied across a conducting wire of radius 1.0 mm. If the resistivity of the conductor is $18.7 \times 10^{-8} \Omega \cdot \text{m}$, how much charge passes through the wire in 10 minutes? [3 points]

$$E = \rho J \Rightarrow J = \frac{E}{\rho} = \frac{6.0 \text{ V}}{18.7 \times 10^{-8} \Omega \cdot \text{m}} = 3.2 \times 10^7 \text{ A/m}^2$$

$$I = J \cdot A = 3.2 \times 10^7 \frac{\text{A}}{\text{m}^2} \times \pi (0.001 \text{ m})^2 = 100.8 \text{ A}$$

$$Q = I \cdot t = 100.8 \text{ A} \times (10 \times 60 \text{ s}) = 6.05 \times 10^4 \text{ C}$$

5. A car battery with $\varepsilon = 12.0 \text{ V}$ shows a terminal voltage 11.3 V , when connected to a light bulb. The power dissipated in the bulb is 4.7 W . What is the internal resistance r of the battery? **[3 points]**



$$P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(11.3 \text{ V})^2}{4.7 \text{ W}} = 27.2 \Omega$$

$$I = \frac{V}{R} = \frac{11.3 \text{ V}}{27.2 \Omega} = 0.415 \text{ A}$$

$$\varepsilon = V + Ir \Rightarrow r = \frac{\varepsilon - V}{I} = \frac{(12.0 - 11.3) \text{ V}}{0.415 \text{ A}} = 1.69 \Omega$$

6. Find the electric currents I_1 and I_2 .

[5 Points]

Junction rule: $I_3 = I_1 + I_2$ (1)

Loop rule (upper):

$$17 - 11I_1 + I_2 - 10 = 0$$

$$7 - 11I_1 + I_2 = 0 \quad (2)$$

Loop rule (big):

$$17 - 11I_1 - 12I_3 = 0$$

Substitute equation (1) in the above equation

$$17 - 23I_1 - 12I_2 = 0 \quad (3)$$

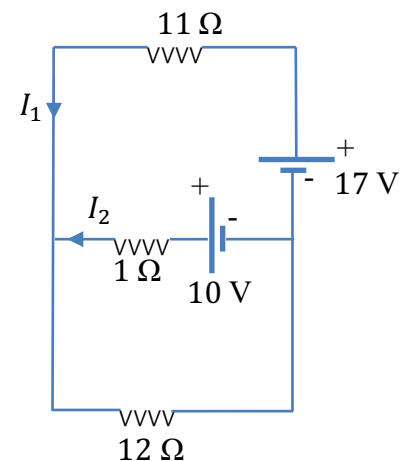
Multiply equation (2) with (+12)

$$(2) \Rightarrow 84 - 132I_1 + 12I_2 = 0 \quad (4)$$

Add equations (3) and (4)

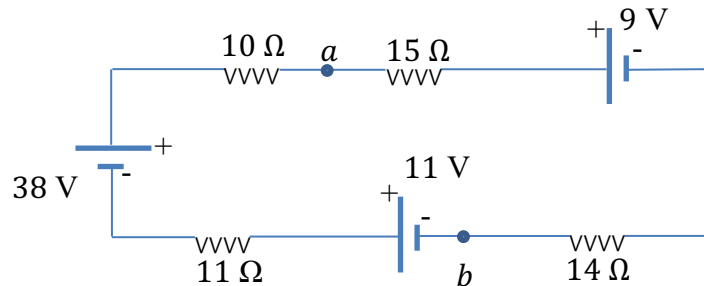
$$101 - 155I_1 = 0 \Rightarrow I_1 = 0.65 \text{ A}$$

$$(2) \Rightarrow I_2 = 0.17 \text{ A}$$



7. Find the potential difference $V_a - V_b$.

[3 Points]



Loop rule

$$38 - 10I - 15I - 9 - 14I + 11 - 11I = 0 \Rightarrow I = 0.8 \text{ A}$$

$$V_a + 10I - 38 + 11I - 11 = V_b \Rightarrow V_{ab} - 49 + 21I = 0$$

$$V_{ab} = 49 - 21I \Rightarrow V_{ab} = 32.2 \text{ V}$$

8. The capacitor is uncharged and at time $t = 0$ the switch is closed. At time t_1 , the electric charge on the capacitor is $q(t_1) = 4 \mu\text{C}$ and when the capacitor is fully charged the charge is $Q = 40 \mu\text{C}$. Find the power dissipated in resistance R_1 at time t_1 .

[4 Points]

Solution 1

$$R_{eq} = R_1 + R_2 \Rightarrow R_{eq} = 10 \Omega$$

$$I_0 = \frac{\varepsilon}{R_{eq}} \Rightarrow I_0 = 2 \text{ A}$$

$$q(t_1) = Q \left[1 - e^{-\frac{t_1}{R_{eq}C}} \right] \Rightarrow \frac{q(t_1)}{Q} = 1 - e^{-\frac{t_1}{R_{eq}C}} \Rightarrow e^{-\frac{t_1}{R_{eq}C}} = 1 - \frac{q(t_1)}{Q} \Rightarrow e^{-\frac{t_1}{R_{eq}C}} = 0.9$$

$$P_{R_1}(t_1) = i^2(t_1)R_1 = I_0^2 R_1 \left(e^{-\frac{t_1}{R_{eq}C}} \right)^2 \Rightarrow P_{R_1}(t_1) = 19.44 \text{ W}$$

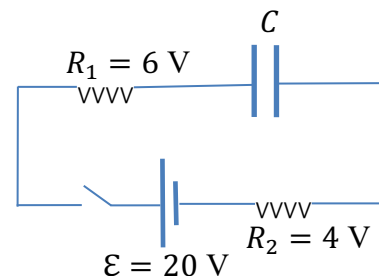
Solution 2

$$R_{eq} = R_1 + R_2 \Rightarrow R_{eq} = 10 \Omega$$

$$C = \frac{Q}{\varepsilon} \Rightarrow C = 2 \mu\text{F}$$

Loop rule: $\varepsilon - i(t_1)R_{eq} - \frac{q(t_1)}{C} = 0 \Rightarrow i(t_1) = 1.8 \text{ A}$.

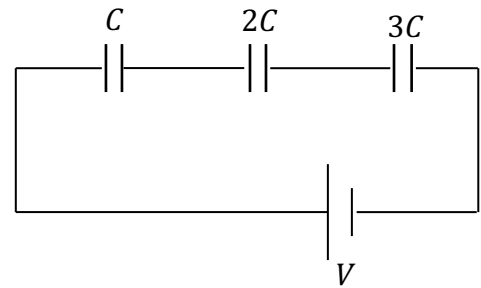
$$P_{R_1}(t_1) = i^2(t_1)R_1 \Rightarrow P_{R_1}(t_1) = 19.44 \text{ W}$$



PART II: Conceptual Questions (each carries 1 point). Tick the best answer:

1. A battery V is connected to three capacitors. The electric charge is

- a) the greatest in capacitor C .
- b) the greatest in capacitor $2C$.
- c) the greatest in capacitor $3C$.
- d) the same in all three capacitors. (ANSWER)

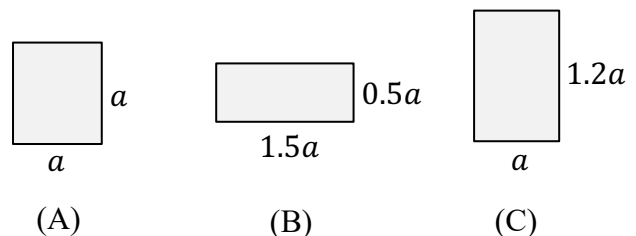


2. An air-filled capacitor C with plate separation d is charged to a potential difference V . The battery is disconnected and then the plate separation is increased to $2d$. Which statement is correct?

- a) The energy stored on the capacitor becomes zero.
- b) The energy stored on the capacitor becomes double. (ANSWER)
- c) The energy stored on the capacitor becomes half.
- d) The energy stored on the capacitor does not change.

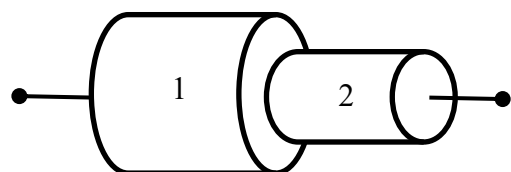
3. The figure shows cross-sections of three conductors of the same material, having the same length. When a potential difference V is applied separately across the length of these, the currents in them will be in the order:

- a) $I_A < I_B < I_C$.
- b) $I_B < I_A < I_C$. (ANSWER)
- c) $I_B < I_C < I_A$.
- d) $I_A < I_C < I_B$.



4. A potential difference is applied across the two ends of a conductor made of two cylinders of the same material with radii $r_1 = 2r_2$. The drift velocity in cylinder 1 is v_d . The drift velocity in cylindrical 2 will be:

- a) v_d .
- b) $2v_d$.
- c) $4v_d$. (ANSWER)
- d) $v_d/4$.



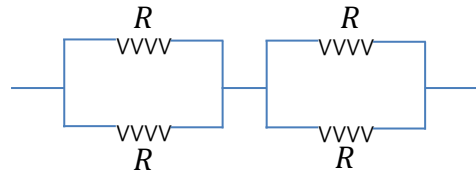
5. In a metallic wire, electrons are moving in the $-x$ direction. Which statement is correct?

- a) The electric current density J has direction $-\hat{i}$ and the applied electric field \vec{E} has $-\hat{i}$.
- b) The electric current density J has direction $-\hat{i}$ and the applied electric field \vec{E} has $+\hat{i}$.
- c) The electric current density J has direction $+\hat{i}$ and the applied electric field \vec{E} has $-\hat{i}$.
- d) The electric current density J has direction $+\hat{i}$ and the applied electric field \vec{E} has $+\hat{i}$.

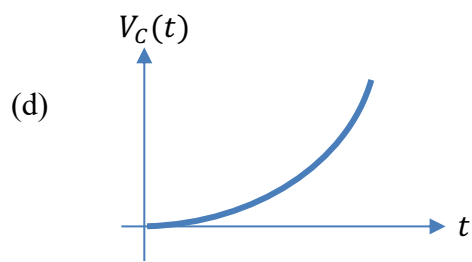
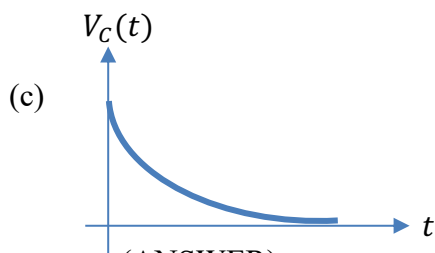
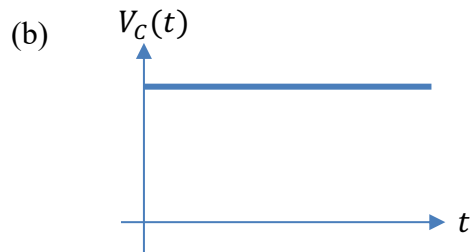
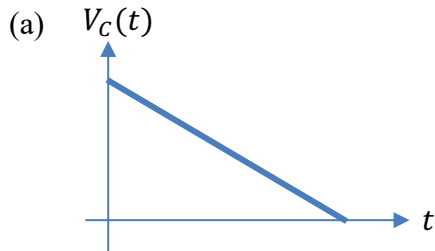
(ANSWER)

6. The equivalent resistance of the network is

- a) $4R$.
- b) $2R$.
- c) R . (ANSWER)
- d) $R/2$.



7. During the discharging of a capacitor, the potential difference $V_C(t)$ across the capacitor versus the time t is described by the diagram



(ANSWER)

8. In the RC circuit below, the switch is closed at time $t = 0$. At time $t = 0$, the electric current is

- a) $I = 0$.
- b) $I = \frac{\epsilon}{R}$. (ANSWER)
- c) $I = \epsilon R$.
- d) $I = C\epsilon$.

