

Askar, Demir, Salameh

# For Instructors use only

Grades:

#	SP1	SP2	SP3	SP4	SP5	LP1	LP2	Q1	8	Q3	Q4	Total
	2	2	2	2	2	3	3	1		1	1	20
Pts								4	$\langle \langle \langle \rangle \rangle$			
								$\overline{\mathbb{Q}}$	$\gamma$			

### Important:

- 1. Answer all questions and proble ( solution = no points).
- 2. Full mark = 20 points as arrang  $\frac{1}{2}$  the above table.
- 3. Give your final answer in the correct units.
- 4. Assume  $g = 10 \text{ m/s}^2$ .
- 5. Mobiles are **<u>strictly prohibited</u>** during the exam.
- 6. Programmable calculators, which can store equations, are not allowed.
- 7. Cheating incidents will be processed according to the university rules.

## GOOD LUCK

#### Part I: Short Problems (2 points each)

SP1. A man made three displacements: from a to b, then from b to c, and finally from c to d, as shown in

the figure. Find the resultant displacement vector  $\vec{R}$  in unit vector notation.

$$\vec{A} = +60 \,\hat{\imath} \, m$$
  

$$\vec{B} = -50 \cos(60^{\circ}) \,\hat{\imath} + 50 \sin(60^{\circ}) \,\hat{\jmath} = (-25\hat{\imath} + 43.3\hat{\jmath}) \, m$$
  

$$\vec{C} = +40 \,\hat{\jmath} \, m$$
  

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} = (35\hat{\imath} + 83.3\hat{\jmath}) \, m$$
  

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} = (35\hat{\imath} + 83.3\hat{\jmath}) \, m$$

**SP2.** A particle moves in the *xy*-plane. Its position vector is given by  $\vec{r}(t) = (3t - 2t^3)\hat{i} + (4t + t^3)\hat{j}$ , where *r* is in meters and *t* is in seconds. Find the <u>magnitude</u> of the particle's acceleration at t = 1s.

$$\vec{v}(t) = \left[ \left( 3 - 6t^2 \right) \hat{i} + \left( 4 + 3t^2 \right) \hat{j} \right] \quad m/s$$
$$\vec{a}(t) = \left( -12t \, \hat{i} + 6t \, \hat{j} \right) \, m/s^2$$
$$\vec{a}(1s) = \left( -12 \, \hat{i} + 6 \, \hat{j} \right) \, m/s^2$$
$$|\vec{a}(1s)| = \sqrt{(12)^2 + (6)^2} = 13.4 \, m/s^2$$

SP3. The velocity-time graph for a runner moving in a straight line is shown. Find the average speed of the runner in the time interval from t = 0 s to t = 10 s.

$$d = area = \frac{1}{2}(3)(8) + (2)(8) + \frac{1}{2}(5)(8) = 48 m$$

$$v_{av} = \frac{d}{t} = \frac{48}{10} = 4.8 m/s$$

$$v_x(m/s)$$

$$v_x(m/s)$$

**SP4.** A stone is thrown straight <u>down</u> from a building of height *h* with an initial speed of 8 m/s, as shown. It takes 0.75 *s* for the stone to hit the ground. Find the height of the building.

$$\Delta y = v_{y_i}t - \frac{1}{2}gt^2$$
$$-h = (-8)(0.75) - \frac{1}{2}(10)(0.75)^2$$
$$h = 8.8 m$$



SP5. A car and a truck start <u>from rest</u> at the same instant, with the car initially at distance d <u>behind</u> the truck, as shown. The truck has a constant acceleration of  $3 m/s^2$ , and the car has an acceleration of  $4 m/s^2$ . The car overtakes the truck after the <u>truck</u> has moved 73.5 m. Find the initial distance (d) between the truck and the car.

For the truck

$$\Delta x = v_{x_i}t + \frac{1}{2}a_xt^2$$
  
73.5 = 0 +  $\frac{1}{2}(3)t^2 \Rightarrow t = 7 s$ 

For the car

$$\Delta x = v_{x_i}t + \frac{1}{2}a_xt^2$$
  
d + 73.5 = 0 +  $\frac{1}{2}(4)7^2 \Rightarrow d = 24.5 m$ 



#### Part II: Long Problems (3 points each)

**LP1:** If 
$$\vec{A} = 2\hat{i} + 3\hat{j} - 2\hat{k}$$
 and  $\vec{B} = \hat{i} - 2\hat{j} + 3\hat{k}$ .  
**a)** Find  $\vec{A} \times \vec{B}$ 

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -2 \\ 1 & -2 & 3 \end{vmatrix}$$
$$= 5i - 8\hat{j} - 7\hat{k}$$

**b**) Find  $\vec{A} \cdot \vec{B}$ 

$$\vec{A} \cdot \vec{B} = (2)(1) + (3)(-2) + (-2)(3) = -10$$

c) Find the angle between  $\vec{A}$  and the positive z-axis.

$$\gamma = \cos^{-1}\left(\frac{A_z}{\left|\vec{A}\right|}\right) = \cos^{-1}\left(\frac{-2}{\sqrt{17}}\right) = 119^o$$

**LP2:** A ball is shot from the ground at an angle of 45° above the horizontal toward a wall, as shown. The ball hits the wall at a point 2 m above the ground level. **Ignore air resistance.** 

# a) How much time does it take for the ball to reach the wall?



$$v_{x_i} = v_i \cos 45^\circ, v_{y_i} = v_i \sin 45^\circ \Rightarrow v_{x_i} = v_{y_i}$$
$$\Delta x = v_{x_i}t \quad \Rightarrow \ t = \frac{\Delta x}{v_{x_i}} = \frac{9.2}{v_{x_i}} = \frac{9.2}{v_{y_i}}$$
$$\Delta y = v_{y_i}t - \frac{1}{2}gt^2$$
$$2 = v_{y_i}(\frac{9.2}{v_{y_i}}) - \frac{1}{2}(10)t^2$$
$$\Rightarrow t = 1.2 s$$

b) Find the initial speed of the ball.

$$\Delta x = v_{x_i} t \Rightarrow v_{x_i} = \frac{\Delta x}{t} = \frac{9.2}{1.2} = 7.67 \ m/s$$
$$v_{x_i} = v_i \cos 45^\circ \Rightarrow v_i = \frac{v_{x_i}}{\cos 45^\circ} = \frac{7.67}{\cos 45^\circ} = 10.8 \ m/s$$

#### Part III: Questions (Choose the correct answer, one point each)

Q1. As a projectile moves along its trajectory. At which point do the velocity and acceleration vectors become **perpendicular** to each other?



**Q2.** If  $\vec{A} = -5\vec{B}$  and  $\vec{C} = \vec{A} \times \vec{B}$ , then \*  $\vec{C} = -6\vec{B}$ \*  $|\vec{C}| = |\vec{A}||\vec{B}|$ \*  $\vec{C} = +5\vec{B}$ (\*)  $\vec{C} = 0$ 

**Q3.** The velocity  $(v_x)$  versus time (*t*) graph for a particle moving along the x-axis is shown in the figure. The velocity and acceleration of the particle at point A, respectively are:

∢ −, +	* +,-

-,- \*+,+



Q4. Which of the following statements is always true?

\* the magnitude of the instantaneous velocity is always greater than the magnitude of the average velocity.

\* the average velocity is always greater than the average speed.

O the average speed can never be negative.

\* the average velocity can never be negative.