



Final Examination
Spring Semester 2023 – 2024

May 22, 2024
Time: 5:00 – 7:00 PM

Name: Student No:

Sec. No: Serial No:

Instructors: Drs. Alaa Alfaiakawi, Peter Lajko, Madan Sharma, and Elias Vagenas

Fundamental constants

- $k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2$ (Coulomb constant)
- $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)$ (Permittivity of free space)
- $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ (Permeability of free space)
- $|e| = 1.60 \times 10^{-19} \text{ C}$ (Elementary unit of charge)
- $N_A = 6.02 \times 10^{23}$ (Avogadro's number)
- $g = 9.8 \text{ m/s}^2$ (Acceleration due to gravity)
- $m_e = 9.11 \times 10^{-31} \text{ kg}$ (Electron mass)
- $m_p = 1.67 \times 10^{-27} \text{ kg}$ (Proton mass)

Prefixes of units

- $m = 10^{-3}$ $\mu = 10^{-6}$ $n = 10^{-9}$ $p = 10^{-12}$
 $k = 10^3$ $M = 10^6$ $G = 10^9$ $T = 10^{12}$

For use by Instructors only

Problems	1	2	3	4	5	6	7	8	9	10	Questions	Total
Marks												

Instructions to the Students:

1. Mobile or other electronic devices are **strictly prohibited** during the exam.
2. Programmable calculators, which can store equations, are not allowed.
3. **Cheating incidents will be processed according to the university rules.**

PART I: Solve the following problems. Show your solutions in detail.

1. Three point charges, $q_1 = q_2 = q_3 = 4 \mu\text{C}$, are placed on the xy -plane, as shown. Calculate the x and y components of the net electric force, \vec{F}_3 , acting on q_3 . **[4 points]**

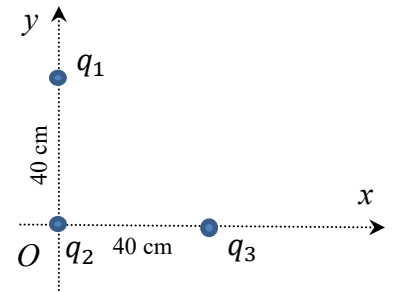
$$r_{13} = \sqrt{(0.4 \text{ m})^2 + (0.4 \text{ m})^2} = 0.57 \text{ m}$$

$$F_{13,x} = k \frac{|q_1 q_3|}{r_{13}^2} \cos(\theta); \quad F_{13,y} = -k \frac{|q_1 q_3|}{r_{13}^2} \sin(\theta);$$

$$F_{23,x} = k \frac{|q_2 q_3|}{r_{23}^2}; \quad F_{23,y} = 0;$$

$$F_{3,x} = F_{13,x} + F_{23,x} = k \frac{|q_1 q_3|}{r_{13}^2} \cos(\theta) + k \frac{|q_2 q_3|}{r_{23}^2} = 1.22 \text{ N}$$

$$F_{3,y} = F_{13,y} + F_{23,y} = -k \frac{|q_1 q_3|}{r_{13}^2} \sin(\theta) + 0 = -0.32 \text{ N}$$

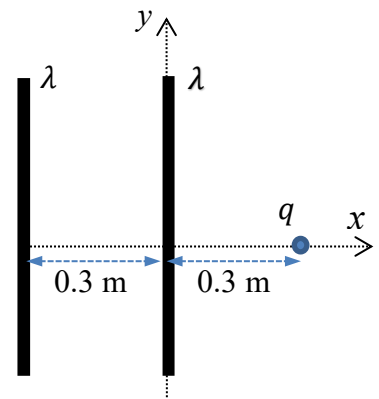


2. Two very long uniformly charged lines, with equal linear charge densities, λ , are placed perpendicular to the x -axis, as shown. If the net force acting on the point charge $q = 3 \mu\text{C}$ is $\vec{F}_{net} = (6 \text{ N})\hat{i}$, find the value of λ . **[4 point]**

$$\vec{E}_1 = \frac{\lambda}{2\pi\epsilon_0 0.6 \text{ m}} \hat{i}$$

$$\vec{E}_2 = \frac{\lambda}{2\pi\epsilon_0 0.3 \text{ m}} \hat{i}$$

$$\vec{F}_{net} = q(\vec{E}_1 + \vec{E}_2) = (6 \text{ N})\hat{i} \Rightarrow \lambda = 22.2 \mu\text{C/m}$$



3. A spherical shell of inner radius $a = 5$ cm and outer radius $b = 10$ cm has uniform volume charge density $\rho = 480$ nC/m³. Find the magnitude and direction of the electric field at 40 cm radial distance from the center. **[3 points]**

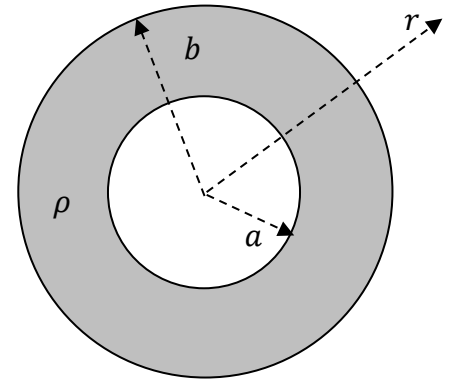
Gauss's Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{encl} = \rho \left(\frac{4}{3} \pi (b^3 - a^3) \right)$$

$$E = \frac{Q_{enc}}{4\pi r^2 \epsilon_0} = 99 \text{ N/C, outward}$$

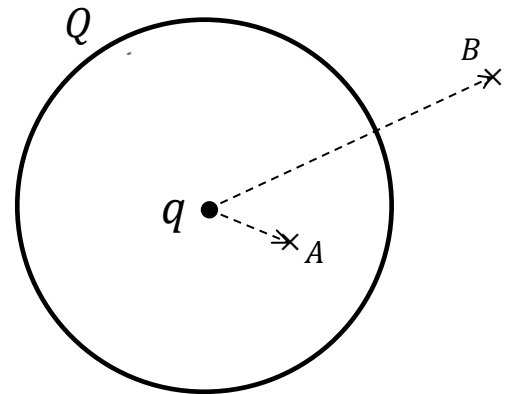


4. A spherical surface of radius $R = 0.5$ m has charge $Q = 2q$ uniformly distributed on its surface and a point charge q is fixed at its centre. If $V = 0$ at infinity, the electric potential at point A is 360 V. What is the electric potential at point B? Given $r_A = 0.25$ m and $r_B = 1.0$ m. **[4 points]**

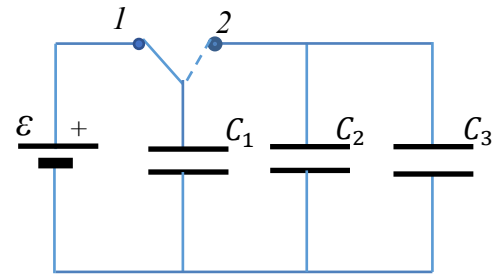
$$V_A = V_{qA} + V_{QA} = k \frac{q}{r_A} + k \frac{2q}{R} = 360 \text{ V} \Rightarrow$$

$$q = 5 \text{ nC}$$

$$V_B = V_{qB} + V_{QB} = k \frac{q}{r_B} + k \frac{2q}{R} = 135 \text{ V}$$



5. A capacitor, $C_1 = 20 \mu\text{F}$, is charged by a battery, $\varepsilon = 24 \text{ V}$, as shown. Then the switch is moved to position 2 so that the capacitor C_1 is connected to uncharged capacitors C_2 and C_3 . If $C_2 = C_3 = 20 \mu\text{F}$, what is the final energy stored in C_3 ? [4 points]



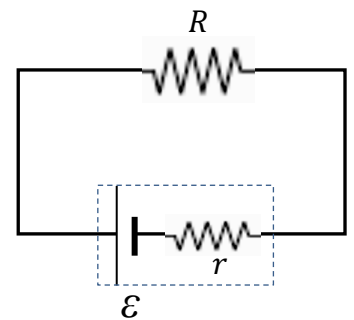
$$Q_0 = \varepsilon C_1 = 480 \mu\text{C}$$

$$C_{eq} = C_1 + C_2 + C_3 = 60 \mu\text{F}$$

$$V^{\text{fin}} = \frac{Q_0}{C_{eq}} = \frac{\varepsilon C_1}{C_1 + C_2 + C_3} = 8 \text{ V}$$

$$U_3^{\text{fin}} = \frac{C_3 (V^{\text{fin}})^2}{2} = 640 \mu\text{J}$$

6. In the circuit shown below, $\varepsilon = 24 \text{ V}$, $r = 1.3 \Omega$, and the terminal voltage of the battery is 21.4 V . Find the power dissipated on resistor R . [4 points]



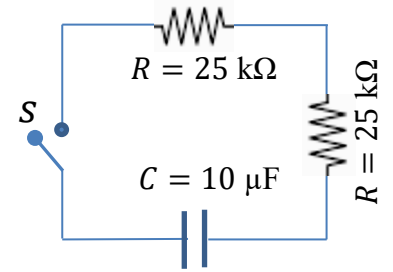
$$V_{ab} = \varepsilon - Ir = 24 \text{ V} - I(1.3 \Omega) = 21.4 \text{ V}$$

$$\Rightarrow I = 2 \text{ A}$$

$$I = \frac{\varepsilon}{r+R} \Rightarrow R = 10.7 \Omega$$

$$P = I^2 R = 42.8 \text{ W}$$

7. In the circuit below, the initial charge on the capacitor is $Q_0 = 40 \text{ nC}$ and the switch is closed at time $t = 0 \text{ s}$. Calculate the time t at which the current in the circuit, I , drops to $1/4$ of its initial value. **[4 points]**



$$\tau = R_{eq}C = 50 \text{ k}\Omega \cdot 10 \text{ }\mu\text{F} = 0.5 \text{ s}$$

$$I(t) = I_0 e^{-t/\tau} \Rightarrow \frac{I_0}{4} = I_0 e^{-t/\tau}$$

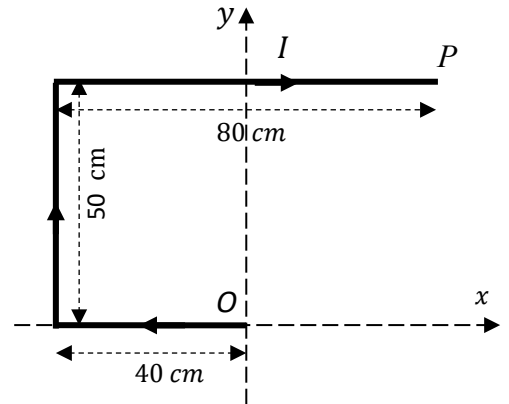
$$t = -R_{eq}C \ln\left(\frac{1}{4}\right) = 0.693 \text{ s}$$

8. A point charge $q = 0.4 \text{ C}$ moves momentarily with velocity $\vec{v} = \left(5 \frac{\text{m}}{\text{s}}\right) \hat{i} + \left(8 \frac{\text{m}}{\text{s}}\right) \hat{j}$ in a region of uniform magnetic field $\vec{B} = (3 \text{ T}) \hat{k}$ and uniform electric field \vec{E} so that the net force acting on the point charge is $\vec{F} = (20 \text{ N}) \hat{i} + (16 \text{ N}) \hat{j}$. Calculate the electric field \vec{E} . **[4 points]**

$$\vec{F}_{net} = q\vec{E} + q\vec{v} \times \vec{B} \Rightarrow \vec{E} = \frac{\vec{F}_{net} - q\vec{v} \times \vec{B}}{q}$$

$$\vec{E} = 26 \frac{\text{N}}{\text{C}} \hat{i} + 55 \frac{\text{N}}{\text{C}} \hat{j}$$

9. A current $I = 7.5 \text{ A}$ flows in a wire from the origin to point P , as shown. Calculate the magnetic force vector acting on the wire by a uniform magnetic field $\vec{B} = (6.0 \text{ T})\hat{k}$. [3 points]

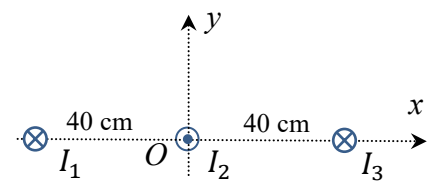


$$\vec{L}_{eff} = -0.4 \text{ m}\hat{i} + 0.5 \text{ m}\hat{j} + 0.8 \text{ m}\hat{i} = 0.4 \text{ m}\hat{i} + 0.5 \text{ m}\hat{j}$$

$$\vec{F} = I \vec{L}_{eff} \times \vec{B} = I[(0.4 \text{ m}\hat{i}) + (0.5 \text{ m}\hat{j})] \times (6.0 \text{ T})\hat{k}$$

$$\vec{F} = (22.5 \text{ N})\hat{i} - (18.0 \text{ N})\hat{j}$$

10. Three very long, parallel wires are perpendicular to the xy -plane and carry currents of magnitude $I_1 = I_2 = I_3 = 8 \text{ A}$ in the directions, as shown. Calculate the net force acting on a 6-m length on the wire of I_3 . [3 points]



$$\vec{F}_{13} = \frac{\mu_0 I_1 I_2 L}{2\pi 0.8 \text{ m}} = -(9.6 \times 10^{-5} \text{ N})\hat{i}$$

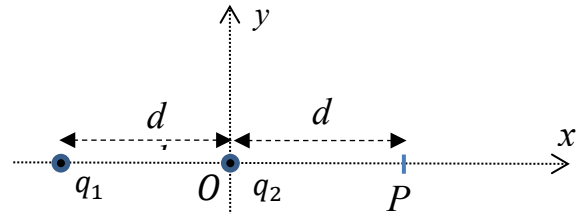
$$\vec{F}_{32} = \frac{\mu_0 I_1 I_3 L}{2\pi 0.4 \text{ m}} = +(19.2 \times 10^{-5} \text{ N})\hat{i}$$

$$\vec{F}_3 = -(9.6 \times 10^{-5} \text{ N})\hat{i} + (19.2 \times 10^{-5} \text{ N})\hat{i} = (9.6 \times 10^{-5} \text{ N})\hat{i}$$

PART II: Conceptual Questions (each carries 1 point). Tick the best answer:

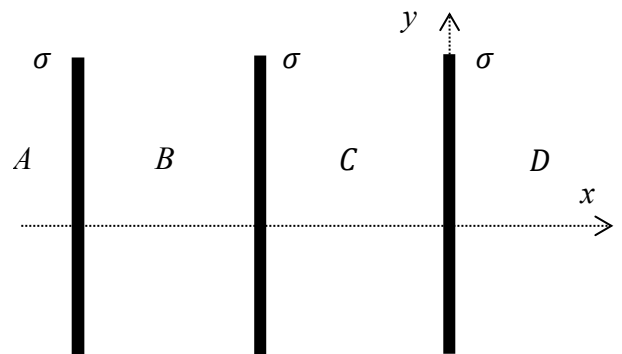
1. Point charge q_1 is fixed on the x -axis. Point charge q_2 is released from point O and moves to point P , as shown. If the electric force on q_2 is \vec{F} at point P , at point O it was

- a) $-\vec{F}$.
- b) \vec{F} .
- c) $4\vec{F}$.
- d) $-4\vec{F}$.



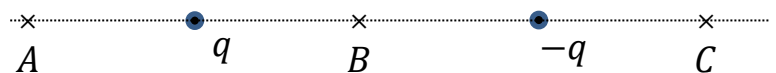
2. Three parallel infinite large planes have uniform surface charge densities σ . In which region is the magnitude of the electric field smallest?

- a) $A \ \& \ B$.
- b) $B \ \& \ C$.
- c) $C \ \& \ D$.
- d) $A \ \& \ D$.



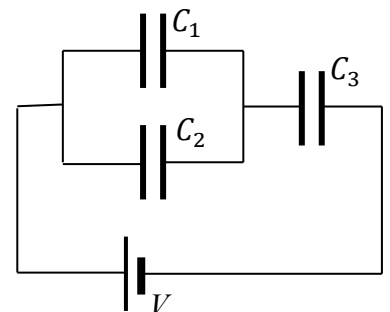
3. Three points A, B , and C , are located around two point charges ($q > 0$), as shown. If $V = 0$ at infinity, at which of these points is the electric potential smallest?

- a) A .
- b) B .
- c) C .
- d) All the points are on the same potential.



4. Three different capacitors are connected into a network as shown. Which statement is correct?

- a) The potential on C_1 is the same as on C_2 .
- b) The potential on C_1 is the same as on C_3 .
- c) The charge of C_1 is the same as of C_2 .
- d) The charge of C_1 is the same as of C_3 .



5. Cylindrical resistor A has length L , radius r and resistance R_A . Cylindrical resistor B is made of the same material, it has length $2L$, radius $2r$, and resistance R_B . Which relation is correct?

a) $R_A = R_B$.

b) $R_A = 2R_B$. ←

c) $R_A = R_B/2$.

d) $R_A = 4R_B$.

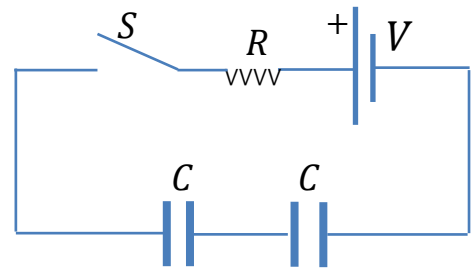
6. In the R - C circuit below, the switch is closed at time $t = 0$ s. After waiting a long time, the electric energy on the left capacitor is

a) $U_L = CV^2$.

b) $U_L = \frac{CV^2}{2}$.

c) $U_L = \frac{CV^2}{4}$.

d) $U_L = \frac{CV^2}{8}$. ←



7. A charged particle moves in a region of uniform magnetic field on a helical path. If the pitch of the helical path is larger than its radius, which relation is true for the parallel and perpendicular components of the velocity?

a) $v_{\parallel} < v_{\perp}$.

b) $v_{\parallel} > v_{\perp}$.

c) $2\pi v_{\parallel} < v_{\perp}$.

d) $2\pi v_{\parallel} > v_{\perp}$. ←

8. A wire carries a current, $I = 4$ A, along the y -axis as shown in the figure. At which point does the magnetic field show along the positive z -axis?

a) At point A .

b) At point B . ←

c) At point A and point B .

d) At none of these points.

