

For Instructors use only

Grades:

#	SP1	SP2	SP3	SP4	SP5	SP6	SP7	LP1	LP2	LP3	Q1	Q2	V _{Q4}	Total
	3	3	3	3	3	3	3	5	5	5	1	707	1	40
Pts												0		

Important:

- 1. Answer all questions and problems (No solution = no points).
- 2. Full mark = 40 points as arranged in the above table.
- 3. Give your final answer in the correct units.
- 4. Assume $g = 10 \text{ m/s}^2$.
- 5. Mobiles are **<u>strictly prohibited</u>** during the exam.
- 6. Programmable calculators, which can store equations, are not allowed.
- 7. Cheating incidents will be processed according to the university rules.

GOOD LUCK

Part I: Short Problems (3 points each)

SP1. The position of a particle moving along the *x*-axis is given by $x(t) = (21 + 22t - 6t^2)$, where x is in m and t is in s. What is the average velocity during the time interval, t = 1 s to t = 3 s?

$$v_{av-x} = \frac{\Delta x}{\Delta t} = \frac{x(3) - x(1)}{3 - 1}$$
$$v_{av-x} = \frac{[21 + 22(3) - 6(3^2)] - [21 + 22(1) - 6(1^2)]}{2}$$
$$= -2 m/s$$

SP2. A force \vec{F} is applied on a block of mass m = 2 kg that is resting on a **rough** horizontal surface ($\mu_s = 0.4$), as shown. Find the minimum value of $|\vec{F}|$ which is required to just make the block slide.

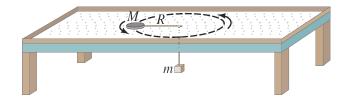


$$F - f_{s,max} = 0$$

$$F = f_{s,max} = \mu_s mg = 0.4 \times 20 = 8 N$$

SP3. A small disk of mass M = 0.3 kg is rotating in a circle of radius R = 0.6 m on a **frictionless horizontal** table with a constant speed of v = 6 m/s. The disk is connected by a light string to a suspended block of mass *m* through a central hole, as shown. **The block** *m* **remains at rest. Find the mass of the suspended block (***m***).**

$$T = mg$$
$$T = \frac{Mv^2}{R}$$
$$m = \frac{Mv^2}{Rg} = \frac{0.3 \times 6^2}{0.6 \times 10} = 1.8 \ kg$$

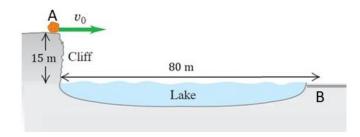


SP4. A rock is projected **horizontally** from the top of a vertical cliff (point A) that is 15 *m* above the surface of a lake, as shown. Find the speed (v_0) of the rock at point A so that it touches the ground at point B.

$$\Delta y = v_{y_i}t - \frac{1}{2}gt^2$$

-15 = 0 - 5t²
$$t = 1.7 s$$

$$v_o = v_{x_i} = \frac{\Delta x}{t} = \frac{80}{1.7} = 46.2 m/s$$



SP5. A wheel rotates through 6 *rad* in 2 *s* as it is being brought to **rest** with **constant** angular acceleration. Find the wheel's initial angular speed (ω_i).

$$\omega_{av} = \frac{\omega_i + \omega_f}{\Delta t}$$

$$\frac{6}{2} = \frac{\omega_i + 0}{2}$$

$$\omega_i = 6 \ rad/s$$

$$OR$$

$$\omega_f = \omega_i + \alpha t$$

$$0 = \omega_i + \alpha(2) \Rightarrow \alpha = \frac{-\omega_i}{2}$$

$$\Delta \theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$6 = \omega_i(2) + \frac{1}{2} (\frac{-\omega_i}{2})(2^2) \Rightarrow \omega_i = 6 \ rad/s$$

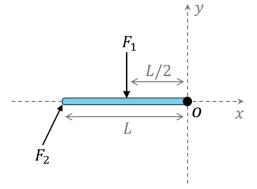


SP6. Two forces, $\vec{F}_1 = -2\hat{j}$ N and $\vec{F}_2 = (3\hat{i} + 6\hat{j})$ N, are applied on a **massless rod** with a length of L = 1.2 m, as shown. Calculate the net torque vector in unit vector notation about point *O* due to these forces.

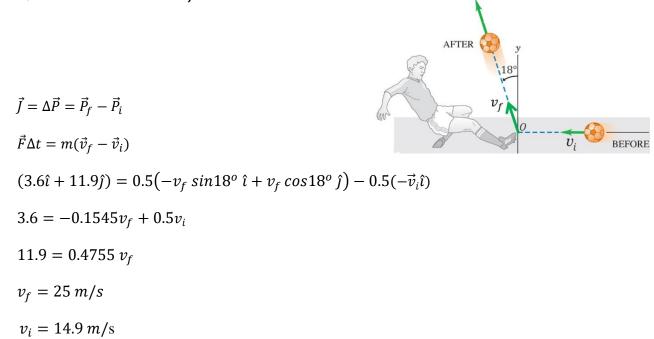
$$\vec{\tau} = -6 \times 1.2 + 2 \times 0.6 = -6 \hat{k} N.m$$

OR

$$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 = -0.6 \ \hat{\imath} \times -2\hat{\jmath} = +1.2 \ \hat{k} \ N. m$$
$$\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2 = -1.2 \ \hat{\imath} \times (3\hat{\imath} + 6\hat{\jmath}) = -7.2 \ \hat{k} \ N. m$$
$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 = -6 \ \hat{k} \ N. m$$



SP7. A football has a mass of 0.5 kg. Initially it is moving to the left at v_i , but then it is kicked with an average force $\vec{F}_{av} = (360 \ \hat{\imath} + 1190 \ \hat{\jmath})$ N during a time of 0.01 s. Immediately after the kick, it moves at 18° upward and to the left with speed v_f , as shown. Use the impulse-momentum theorem to find the initial speed (v_i) and the final speed (v_f) of the ball.



Part II: Long Problems (5 points each)

LP1. A block (m = 3 kg) is suspended from the free end of a light rope which is wrapped around a frictionless pulley $(M = 2 kg, R = 0.2 m, I = 0.04 kgm^2)$, as shown. The block is released from **rest** at a distance of 1.5 *m* above the floor.

a) Find the block's speed just before it strikes the floor.

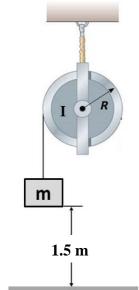
$$E_{i} = E_{f}$$

$$mgh = \frac{1}{2}mv_{f}^{2} + \frac{1}{2}I\omega_{f}^{2}$$

$$mgh = \frac{1}{2}mv_{f}^{2} + \frac{1}{2}I(\frac{v_{f}}{R})^{2}$$

$$30 \times 1.5 = \frac{1}{2} \times 3 \times v_{f}^{2} + \frac{1}{2} \times 0.04 \times (\frac{v_{f}}{0.2})^{2}$$

$$v_{f} = 4.74 \text{ m/s}$$



b) Find the **angular speed of the pulley** just before the block strikes the floor.

$$\omega_f = \frac{v_f}{R} = \frac{4.74}{0.2} = 23.7 \, m/s$$

- c) During this motion, the <u>total</u> work done by the tension on the system of the block and the pulley is:
 - * positive

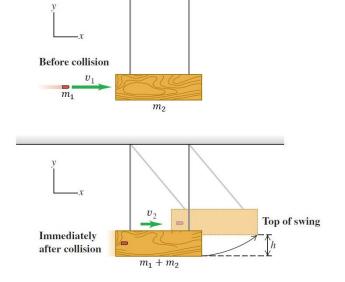
* negative



LP2. A bullet of mass of $m_1 = 50 \ g$ moves horizontally with a speed of $v_1 = 250 \text{ m/s}$ makes a completely inelastic collision with a block of wood of mass $m_2 = 4 \text{ kg}$, which is suspended like a pendulum. After the impact, the system swings up to <u>a maximum</u> height *h*, as shown.

a) What is the momentum of the system **immediately after the collision?**

$$P_{xf} = P_{xi} = m_1 v_{xi} = 0.05(250) = 12.5 \ kg. \ m/s$$



b) What is the speed of the system **immediately after the collision**?

$$P_{xi} = P_{xf}$$

$$m_1 v_1 = (m_1 + m_2) v_2$$

$$v_2 = \frac{m_1 v_1}{(m_1 + m_2)} = \frac{12.5}{4.05} = 3.09 \text{ m/s}$$

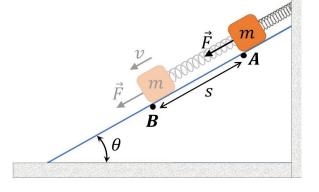
c) Find the maximum height (*h*).

$$\frac{1}{2}(m_1 + m_2)v_2^2 = (m_1 + m_2)gh$$
$$h = \frac{v_2^2}{2g} = 0.48 m$$

LP3. A block of mass (m = 0.4 kg) is attached to a light spring (k = 100 N/m) on a **rough** inclined plane $(\theta = 30^\circ, \mu_k = 0.3)$. An applied force $(|\vec{F}| = 30 N)$ acts on the block, as shown. The block is **lowered down** a distance of (s = 0.4 m) from point **A** to point **B**. At point **A**, the spring is relaxed (x = 0).

a) How much work is done on the block by the force \vec{F} from A to B?

$$W_F = \vec{F} \cdot \vec{S} = |\vec{F}| |\vec{S}| \cos(0) = 30 \times 0.4 = 12 J$$



b) How much work is done on the block by the force of friction from A to B?

$$W_{f_k} = \overrightarrow{f_k}. \vec{S} = |\overrightarrow{f_k}| |\vec{S}| \cos(180) = -\mu_k mg \cos(\theta) \times 0.4$$
$$W_{f_k} = -0.3 \times 4 \times 0.87 \times 0.4 = -0.42 J$$

c) What is the **change in the block's kinetic energy** as it moves from A to B?

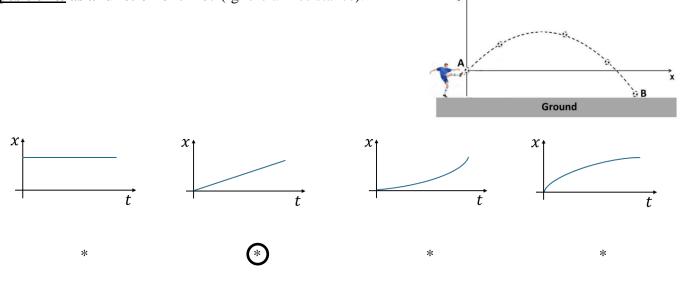
$$\Delta K = W_{tot} = W_F + W_{f_k} + W_{el} + W_g + W_n$$

$$\Delta K = 12 - 0.42 - \frac{1}{2}kx^2 + mgd\sin(\theta) + 0$$

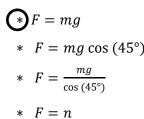
$$\Delta K = 11.6 - 0.5 \times 100 \times (0.4)^2 + 4 \times 0.4 \times 0.5 + 0 = 4.4J$$

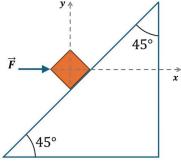
Part III: Questions (Choose the correct answer, one point each)

Q1. A ball is shot such that it leaves the player's foot at point A above ground level, as shown. During the ball's travel from point A to point B, which of the following graphs represents the ball's <u>horizontal</u> <u>position x as a function of time?</u> (Ignore air resistance). y_{\uparrow}



Q2. A force \vec{F} is exerted horizontally on a box of mass m, as shown. The force keeps the box at rest. The incline is frictionless. Which of the following is correct:





Q3. A spring of force constant k_1 is stretched a certain distance (x_1) . It requires the same work to stretch a second spring, with spring constant k_2 , by a distance $(x_2 = \frac{1}{2}x_1)$. Which of the following is correct:

Q4. In a collision between two masses $(m_1 < m_2)$, which mass receives an impulse with greater magnitude?

* m_1 • m_1 and m_2 receive impulses of equal magnitude. * m₂

* It depends on their initial velocities.