

For Instructors use only

Grades:

Important:

- 1. Answer all questions and problems (No solution = no points).
- 2. Full mark = 40 points as arranged in the above table.
- 3. **Give your final answer in the correct units.**
- 4. Assume $g = 10 \text{ m/s}^2$.
- 5. Mobiles are **strictly prohibited** during the exam.
- 6. Programmable calculators, which can store equations, are not allowed.
- 7. **Cheating incidents will be processed according to the university rules.**

GOOD LUCK

Part I: Short Problems (3 points each)

SP1. The position of a particle moving along the *x-axis* is given by $x(t) = (21 + 22t - 6t^2)$, where *x* is in *m* and *t* is in s. What is the average velocity during the time interval, $t = 1$ s to $t = 3$ s?

$$
v_{av-x} = \frac{\Delta x}{\Delta t} = \frac{x(3) - x(1)}{3 - 1}
$$

$$
v_{av-x} = \frac{[21 + 22(3) - 6(3^{2})] - [21 + 22(1) - 6(1^{2})]}{2}
$$

$$
= -2 \, m/s
$$

SP2. A force \vec{F} is applied on a block of mass $m = 2$ kg that is resting on a **rough** horizontal surface (μ_s = 0.4), as shown. **Find the minimum value of** $|\vec{F}|$ which is required to just make the block slide.

$$
F - f_{s,max} = 0
$$

$$
F = f_{s,max} = \mu_s mg = 0.4 \times 20 = 8 N
$$

SP3. A small disk of mass $M = 0.3$ kg is rotating in a circle of radius $R = 0.6$ m on a **frictionless horizontal** table with a constant speed of $v = 6$ m/s. The disk is connected by a light string to a suspended block of mass *m* through a central hole, as shown. The block *m* remains at rest. Find the mass of the suspended **block ()**.

$$
T = mg
$$

\n
$$
T = \frac{Mv^2}{R}
$$

\n
$$
m = \frac{Mv^2}{Rg} = \frac{0.3 \times 6^2}{0.6 \times 10} = 1.8 kg
$$

SP4. A rock is projected **horizontally** from the top of a vertical cliff (point A) that is $15 \, \text{m}$ above the surface of a lake, as shown. **Find the speed** (v_0) **of the rock at point A** so that it touches the ground at point B.

$$
\Delta y = v_{y_i} t - \frac{1}{2} g t^2
$$

-15 = 0 - 5t²

$$
t = 1.7 s
$$

$$
v_o = v_{x_i} = \frac{\Delta x}{t} = \frac{80}{1.7} = 46.2 m/s
$$

SP5. A wheel rotates through 6 *rad* in 2 *s* as it is being brought to **rest** with **constant** angular acceleration. Find the wheel's initial angular speed (ω_i) .

$$
\omega_{av} = \frac{\omega_i + \omega_f}{\Delta t}
$$

\n
$$
\frac{6}{2} = \frac{\omega_i + 0}{2}
$$

\n
$$
\omega_i = 6 \text{ rad/s}
$$

\nOR
\n
$$
\omega_f = \omega_i + \alpha t
$$

\n
$$
0 = \omega_i + \alpha (2) \Rightarrow \alpha = \frac{-\omega_i}{2}
$$

\n
$$
\Delta \theta = \omega_i t + \frac{1}{2} \alpha t^2
$$

\n
$$
6 = \omega_i (2) + \frac{1}{2} (\frac{-\omega_i}{2}) (2^2) \Rightarrow \omega_i = 6 \text{ rad/s}
$$

SP6. Two forces, $\vec{F}_1 = -2\hat{j}$ N and $\vec{F}_2 = (3\hat{i} + 6\hat{j})$ N, are applied on a **massless rod** with a length of $L =$ 1.2 m , as shown. **Calculate the net torque vector in unit vector notation about point** O **due to these forces.**

$$
\vec{\tau} = -6 \times 1.2 + 2 \times 0.6 = -6 \,\hat{k} \, N.m
$$

OR

$$
\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 = -0.6 \hat{\imath} \times -2\hat{\jmath} = +1.2 \hat{k} \text{ N} \cdot \text{m}
$$

$$
\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2 = -1.2 \hat{\imath} \times (3\hat{\imath} + 6\hat{\jmath}) = -7.2 \hat{k} \text{ N} \cdot \text{m}
$$

$$
\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 = -6 \hat{k} \text{ N} \cdot \text{m}
$$

SP7. A football has a mass of 0.5 kg. Initially it is moving to the left at v_i , but then it is kicked with an average force $\vec{F}_{av} = (360 \hat{i} + 1190 \hat{j})$ N during a time of 0.01 s. Immediately after the kick, it moves at 18° upward and to the left with speed v_f , as shown. **Use the impulse-momentum theorem to find the initial** $\mathbf{s}\mathbf{p}\mathbf{e}\mathbf{e}\mathbf{d}\left(\mathbf{v}_{i}\right)$ and the final $\mathbf{s}\mathbf{p}\mathbf{e}\mathbf{e}\mathbf{d}\left(\mathbf{v}_{f}\right)$ of the ball.

Part II: Long Problems (5 points each)

LP1. A block $(m = 3 kg)$ is suspended from the free end of a light rope which is wrapped around a frictionless pulley ($M = 2$ kg, $R = 0.2$ m, $I = 0.04$ kgm²), as shown. The block is released from **rest** at a distance of 1.5 *above the floor.*

a) Find the block's speed just before it strikes the floor.

$$
E_i = E_f
$$

\n
$$
mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2
$$

\n
$$
mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I(\frac{v_f}{R})^2
$$

\n
$$
30 \times 1.5 = \frac{1}{2} \times 3 \times v_f^2 + \frac{1}{2} \times 0.04 \times (\frac{v_f}{0.2})^2
$$

\n
$$
v_f = 4.74 \, m/s
$$

b) Find the **angular speed of the pulley** just before the block strikes the floor.

$$
\omega_f = \frac{v_f}{R} = \frac{4.74}{0.2} = 23.7 \, m/s
$$

- c) During this motion, the **total work done by the tension** on the system of the block and the pulley is:
	- * positive * negative * negative * \bullet zero

LP2. A bullet of mass of $m_1 = 50$ g moves horizontally with a speed of $v_1 = 250$ m/s makes **a completely inelastic collision** with a block of wood of mass $m_2 = 4$ kg, which is suspended like a pendulum. After the impact, the system swings up to **a maximum height** ℎ, as shown.

a) What is the momentum of the system **immediately after the collision?**

$$
P_{xf} = P_{xi} = m_1 v_{xi} = 0.05(250) = 12.5 kg \cdot m/s
$$

b) What is the speed of the system **immediately after the collision**?

$$
P_{xi} = P_{xf}
$$

\n
$$
m_1 v_1 = (m_1 + m_2)v_2
$$

\n
$$
v_2 = \frac{m_1 v_1}{(m_1 + m_2)} = \frac{12.5}{4.05} = 3.09 \, m/s
$$

c) Find the maximum height (h) .

$$
\frac{1}{2}(m_1 + m_2)v_2^2 = (m_1 + m_2)gh
$$

$$
h = \frac{v_2^2}{2g} = 0.48 m
$$

LP3. A block of mass ($m = 0.4$ kg) is attached to a light spring ($k = 100$ N/m) on a **rough** inclined plane $(\theta = 30^\circ, \mu_k = 0.3)$. An applied force $(|\vec{F}| = 30 \text{ N})$ acts on the block, as shown. The block is **lowered down** a distance of $(s = 0.4 \text{ m})$ from point **A** to point **B**. At point **A**, the spring is relaxed $(x = 0)$.

a) How much work is done on the block **by the force** \vec{F} from A to B?

$$
W_F = \vec{F} \cdot \vec{S} = |\vec{F}| |\vec{S}| \cos(0) = 30 \times 0.4 = 12 \text{ J}
$$

$$
W_{f_k} = \overrightarrow{f_k} \cdot \overrightarrow{S} = |\overrightarrow{f_k}| |\overrightarrow{S}| \cos(180) = -\mu_k mg \cos(\theta) \times 0.4
$$

$$
W_{f_k} = -0.3 \times 4 \times 0.87 \times 0.4 = -0.42 J
$$

c) What is the **change in the block's kinetic energy** as it moves from A to B?

$$
\Delta K = W_{tot} = W_F + W_{f_k} + W_{el} + W_g + W_n
$$

\n
$$
\Delta K = 12 - 0.42 - \frac{1}{2}kx^2 + mgd \sin(\theta) + 0
$$

\n
$$
\Delta K = 11.6 - 0.5 \times 100 \times (0.4)^2 + 4 \times 0.4 \times 0.5 + 0 = 4.4 J
$$

Part III: Questions (Choose the correct answer, one point each)

Q1. A ball is shot such that it leaves the player's foot at point A above ground level, as shown. During the ball's travel from point A to point B, **which of the following graphs represents the ball's horizontal position** x **as a function of time?** (Ignore air resistance).

Q2. A force \vec{F} is exerted horizontally on a box of mass m, as shown. The force keeps the box at rest. The incline is **frictionless. Which of the following is correct:**

Q3. A spring of force constant k_1 is stretched a certain distance (x_1) . It requires the same work to stretch a second spring, with spring constant k_2 , by a distance $(x_2 = \frac{1}{2})$ $\frac{1}{2}x_1$). Which of the following is correct:

*
$$
k_2 = k_1
$$
 * $k_2 = 2k_1$ * $k_2 = 4k_1$ * $k_2 = 8k_1$ * $k_2 = 8k_1$

Q4. In a collision between two masses $(m_1 < m_2)$, which mass receives **an impulse with greater magnitude?**

* m_1 * m_2 $\bigotimes m_1$ and m_2 receive impulses of equal magnitude. * It depends on their initial velocities.