



Final Examination
Fall Semester 2024 – 2025

December 29, 2024
Time: 2:00 – 4:00 PM

Name: Student No: Sec. No: ... Serial No:

Instructors: Drs. Abdulla, Almumin, Lajko, Sharma, and Vagenas

Fundamental constants

$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2$	(Coulomb constant)
$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)$	(Permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$	(Permeability of free space)
$ e = 1.60 \times 10^{-19} \text{ C}$	(Elementary unit of charge)
$N_A = 6.02 \times 10^{23}$	(Avogadro's number)
$g = 9.8 \text{ m/s}^2$	(Acceleration due to gravity)
$m_e = 9.11 \times 10^{-31} \text{ kg}$	(Electron mass)
$m_p = 1.67 \times 10^{-27} \text{ kg}$	(Proton mass)

Prefixes of units

$m = 10^{-3}$	$\mu = 10^{-6}$	$n = 10^{-9}$	$p = 10^{-12}$
$k = 10^3$	$M = 10^6$	$G = 10^9$	$T = 10^{12}$

For use by Instructors only

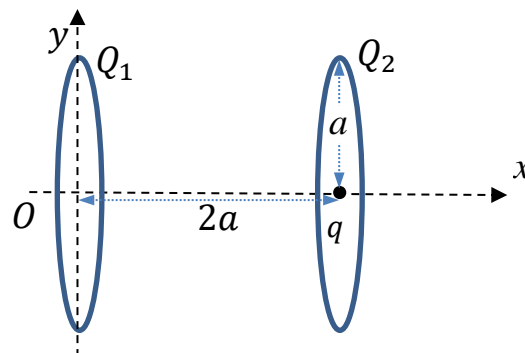
Problems	1	2	3	4	5	6	7	8	9	10	Questions	Total
Marks												

Instructions to the Students:

1. Mobile or other electronic devices are **strictly prohibited** during the exam.
2. Programmable calculators, which can store equations, are not allowed.
3. **Cheating incidents will be processed according to the university rules.**

PART I: Solve the following problems. Show your solutions in detail.

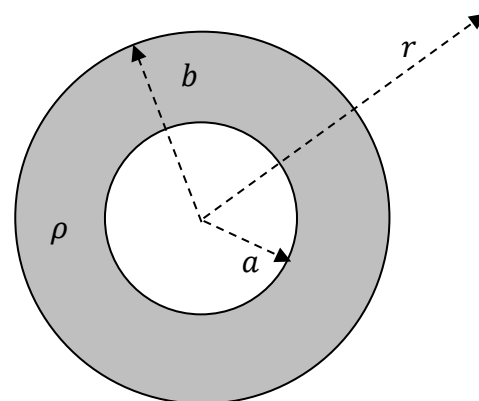
1. Two rings of identical radii, a , have charges $Q_1 = -4 \mu\text{C}$ and $Q_2 = 4 \mu\text{C}$, as shown, uniformly distributed along them. A point charge $q = 6 \mu\text{C}$ is placed at the center point of the second ring. If $a = 0.4 \text{ m}$, determine the net electric force vector \vec{F} acting on q . [3 points]



$$\begin{aligned}\vec{E}_{\text{ring}_1} &= k \frac{|Q_1|2a}{((2a)^2+a^2)^{3/2}} (-\hat{i}) \\ &= -\frac{9 \times 10^9 \times 4.0 \times 10^{-6} \times 0.8}{(0.8^2+0.40^2)^{3/2}} \hat{i} \text{ N/C} \\ &= -4.0 \times 10^4 \hat{i} \text{ N/C}\end{aligned}$$

$$\begin{aligned}\vec{F} &= q\vec{E}_1 = 6.0 \times 10^{-6} \times (-4.0) \times 10^4 \hat{i} \text{ N} \\ &= -0.24 \hat{i} \text{ N}\end{aligned}$$

2. A spherical shell of inner radius $a = 8 \text{ cm}$ and outer radius $b = 16 \text{ cm}$ has uniform volume charge density $\rho = 300 \text{ nC/m}^3$. Determine the magnitude and direction of the net electric field at a radial distance 24 cm from the center. [3 points]



Gauss's Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$Q_{\text{encl}} = \rho \left(\frac{4}{3} \pi (b^3 - a^3) \right) = 4.5 \text{ nC}$$

$$E = \frac{Q_{\text{enc}}}{4\pi r^2 \epsilon_0} = 703.7 \text{ N/C, outward}$$

3. A thin spherical shell of radius $R = 0.5$ m has uniform surface charge density $\sigma = 90$ nC/m². Calculate the electric potential difference, $V_A - V_B$, between points A and B . Given $r_A = 0.25$ m and $r_B = 1.5$ m.

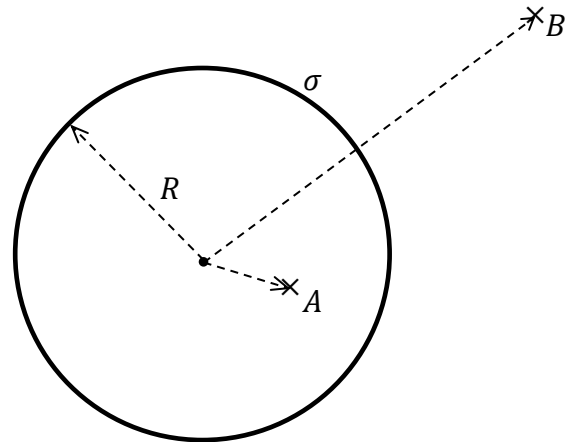
[4 points]

$$Q = \sigma 4\pi R^2 = 283 \text{ nC}$$

$$V_A = \frac{kQ}{R} = k\sigma 4\pi R = 5089.4 \text{ V}$$

$$V_B = \frac{kQ}{r_B} = 1696.5 \text{ V}$$

$$V_A - V_B = 3393 \text{ V}$$



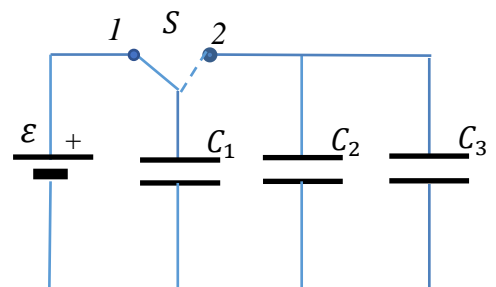
4. A capacitor, $C_1 = 20$ μF , is charged by a battery, as shown. Then, the switch is moved to position 2 so that capacitor C_1 is connected to the uncharged capacitors $C_2 = C_3 = 10$ μF , as shown. If the final energy stored in C_3 is $U_3^{fin} = 45$ μJ , determine the value of ε .

[4 points]

$$U_3^{fin} = \frac{C_3 V^2}{2} = 45 \text{ } \mu\text{J} \Rightarrow V = 3 \text{ V}$$

$$Q_0 = Q_{fin} = (C_1 + C_2 + C_3)V = 120 \text{ } \mu\text{C}$$

$$\text{so } V = \frac{Q_0}{C_1 + C_2 + C_3} = \frac{C_1 \varepsilon}{C_1 + C_2 + C_3} \Rightarrow \varepsilon = 6 \text{ V}$$

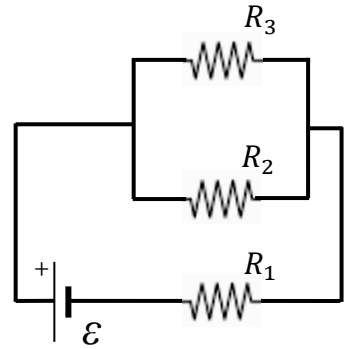


5. When an ideal emf is connected to a network of resistors, as shown, the power dissipated in R_2 is 32 W. If $R_1 = R_2 = R_3 = 8 \Omega$, determine the value of \mathcal{E} . [4 points]

$$P = I_2^2 R_2 = 32 \text{ W} \Rightarrow I_2 = 2 \text{ A}$$

$$\text{From Junction rule: } I_1 = I_2 + I_3 = 4 \text{ A}$$

$$\text{Loop rule: } \mathcal{E} = I_1 R_1 + I_2 R_2 = 48 \text{ V}$$

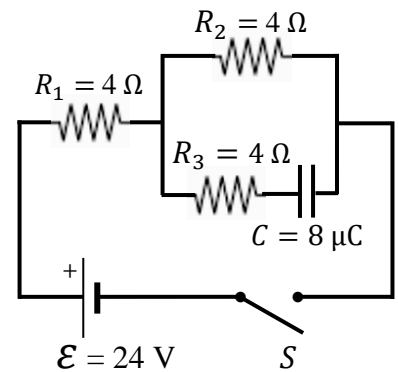


6. In the circuit shown below, the capacitor is initially uncharged and the switch, S , is closed at $t = 0$. What is the current in the emf device a long time after the switch, S , is closed? [3 points]

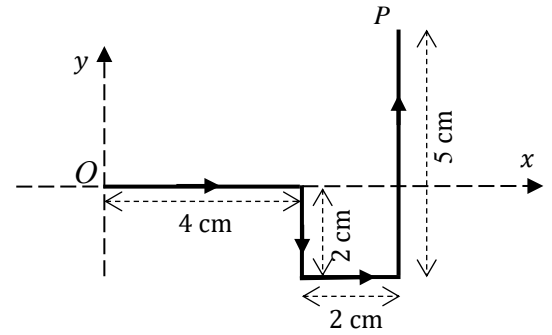
A long time after the switch is closed, the capacitor represents a cut, so no current through it.

$$\Rightarrow R_{eq} = R_1 + R_2 = 8 \Omega$$

$$\text{Loop rule: } I_1 = \frac{\mathcal{E}}{R_{eq}} = 3 \text{ A}$$



7. A wire carries a current of $I = 12.0 \text{ A}$ from point O (origin) to point P , as shown. Calculate the magnetic force vector, \vec{F} , acting on the wire due to a uniform magnetic field $\vec{B} = (-4.0 \text{ T})\hat{k}$. [3 points]



$$\vec{L}_{eff} = (\Delta x)\hat{i} + (\Delta y)\hat{j} = (0.06 \text{ m})\hat{i} + (-0.02 \text{ m} + 0.05 \text{ m})\hat{j}$$

$$\vec{F} = I\vec{L}_{eff} \times \vec{B} = 12.0 \text{ A}[(0.06 \text{ m})\hat{i} + (0.03 \text{ m})\hat{j}] \times [(-4.0 \text{ T})\hat{k}]$$

$$\vec{F} = [-(1.44 \text{ N})\hat{i} + (2.88 \text{ N})\hat{j}]$$

8. A proton moves momentarily with a velocity of $\vec{v} = (3 \times 10^6 \frac{\text{m}}{\text{s}})\hat{i} + (2 \times 10^6 \frac{\text{m}}{\text{s}})\hat{j}$ in a region of uniform magnetic field $\vec{B} = (2 \text{ T})\hat{j}$. Find the radius and pitch of the helical path of the proton. [4 points]

$$v_{\perp} = 3 \times 10^6 \frac{\text{m}}{\text{s}}$$

$$v_{\parallel} = 2 \times 10^6 \frac{\text{m}}{\text{s}}$$

$$R = \frac{mv_{\perp}}{|q|B} = 0.016 \text{ m}$$

$$P = v_{\parallel}T = v_{\parallel} \frac{2\pi m}{|q|B} = 0.066 \text{ m}$$

9. Two very long wires are perpendicular to the xy -plane. The wires carry a current of $I_1 = I_2 = 10$ A, as shown. Calculate the magnetic force vector, \vec{F} , acting on charge $q = 4 \mu\text{C}$ of velocity $\vec{v} = \left(4 \frac{\text{m}}{\text{s}}\right) \hat{i}$.

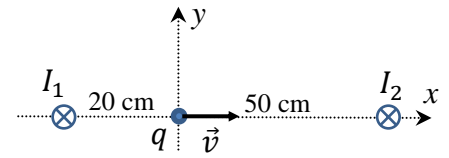
[4 points]

$$\vec{B}_1 = -\frac{\mu_0 I_1}{2\pi 0.2\text{m}} \hat{j} = -(1.0 \times 10^{-5} \text{T}) \hat{j}$$

$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi 0.5\text{m}} \hat{j} = (4.0 \times 10^{-6} \text{T}) \hat{j} \Rightarrow$$

$$\vec{B}_{\text{net}} = -(6 \times 10^{-6} \text{T}) \hat{j}$$

$$\vec{F} = q\vec{v} \times \vec{B}_{\text{net}} = -4 \mu\text{C} \left(4 \frac{\text{m}}{\text{s}}\right) \hat{i} \times (6 \times 10^{-6} \text{T}) \hat{j} = -(9.6 \times 10^{-11} \text{N}) \hat{k}$$

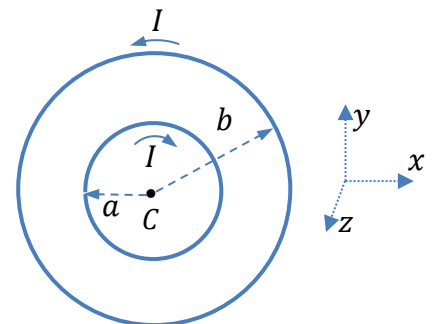


10. Two concentric circular loops of radii, $a = 0.2$ m, $b = 0.5$ m, carry identical currents of $I = 5$ A, as shown. Calculate the magnitude and direction of magnetic field at the center point, C , of the loops. [3 points]

$$\vec{B}_a = -\left(\frac{\mu_0 I}{2a}\right) \hat{k} = -(1.57 \times 10^{-5} \text{T}) \hat{k}$$

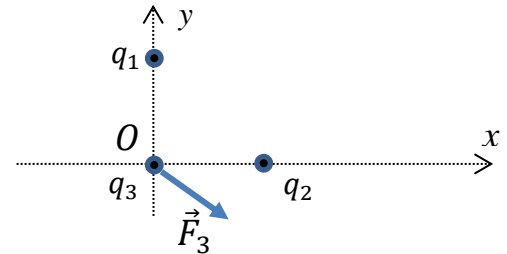
$$\vec{B}_b = \left(\frac{\mu_0 I}{2b}\right) \hat{k} = (6.3 \times 10^{-6} \text{T}) \hat{k}$$

$$\vec{B}_{\text{net}} = \vec{B}_a + \vec{B}_b = -(9.4 \times 10^{-6} \text{T}) \hat{k}$$



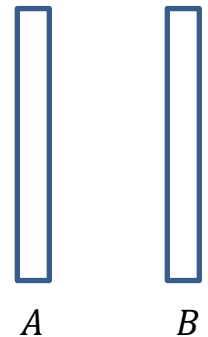
PART II: Conceptual Questions (each carries 1 point). Tick the best answer:

1. Three point charges $q_1, q_2,$ and $q_3,$ are placed on the xy -plane. The net electric force $\vec{F}_3,$ acting on charge q_3 is shown. If $q_3 < 0,$ which statement can be correct for the charges?



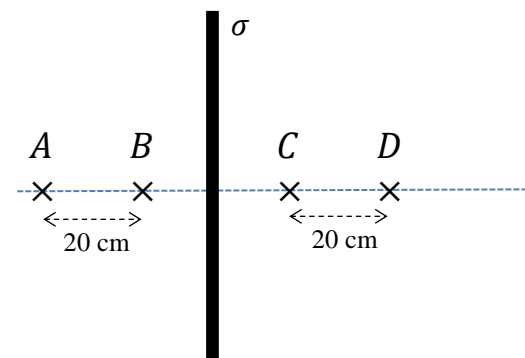
- a) $q_1 < 0$ and $q_2 < 0.$
- b) $q_1 < 0$ and $q_2 > 0.$ ←
- c) $q_1 > 0$ and $q_2 > 0.$
- d) $q_1 > 0$ and $q_2 < 0.$

2. Two very large parallel conducting plates $A,$ and B carry opposite charges $+Q$ and $-Q.$ Which statement is correct for the location of the charges? The charges are



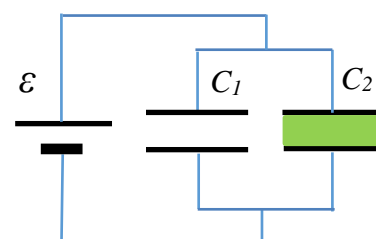
- a) on the right surface of plate A and on the right surface of plate $B.$
- b) on the right surface of plate A and on the left surface of plate $B.$ ←
- c) on the left surface of plate A and on the right surface of plate $B.$
- d) on the left surface of plate A and on the left surface of plate $B.$

3. A large uniformly charged sheet has a uniform surface charge density $\sigma > 0.$ Which relation is correct for the electric potential at point $A, B, C,$ and $D?$



- a) $V_A - V_B = V_C - V_D.$
- b) $V_A - V_B = V_B - V_C.$
- c) $V_A - V_B = V_D - V_C.$ ←
- d) $V_A - V_C = V_C - V_D.$

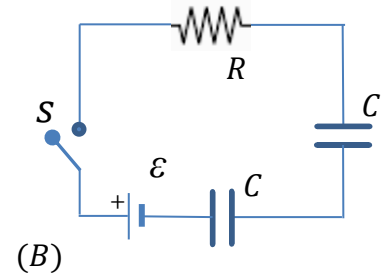
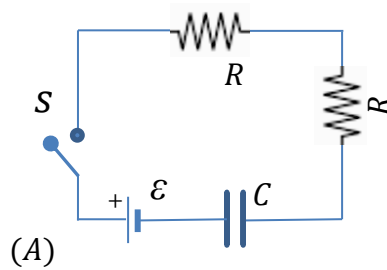
4. Two identical parallel-plate capacitors $C_1 = C_2$ are connected to a battery, and then capacitor C_2 is filled with a dielectric material of $K = 2.$ If the energy density in the capacitors $C_1,$ and C_2 are $u_1,$ and $u_2,$ respectively, which relation is correct?



- a) $u_1 = u_2.$
- b) $u_1 = 2u_2.$
- c) $4u_1 = u_2.$
- d) $2u_1 = u_2.$ ←

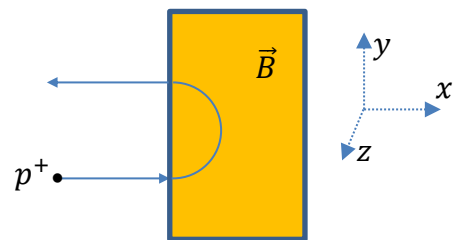
5. Two R - C circuits are shown. If the time constant in circuit A is τ_A , and the time constant in circuit B is τ_B ,

- a) $\tau_A = \tau_B$.
- b) $\tau_A = 2\tau_B$.
- c) $\tau_A = 4\tau_B$.
- d) $2\tau_A = \tau_B$.



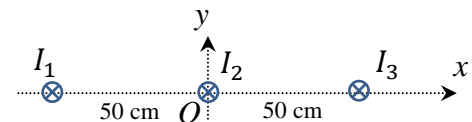
6. A proton enters and leaves a rectangular region of uniform magnetic field on a semicircular path, as shown. The direction of the magnetic field is

- a) \hat{k} .
- b) \hat{j} .
- c) \hat{i} .
- d) $-\hat{k}$.



7. Three very long parallel wires are carrying identical currents $I_1 = I_2 = I_3$, as shown. If the magnitude of the net magnetic force acting on them are F_1, F_2 , and F_3 , respectively, which statement is correct?

- a) $F_1 = F_3$.
- b) $F_2 = F_3$.
- c) $F_1 = F_2$.
- d) $F_1 = F_2 = F_3$.



8. The figure shows a closed curve, C , its orientation and currents I_1, I_2 which are perpendicular to the plane of the curve, as shown. If the value of the curve integral on C is $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$, which relation is correct for the magnitudes of the currents, I_1, I_2 ?

- a) $I_1 = I, I_2 = I$.
- b) $I_1 = I, I_2 = 2I$.
- c) $I_1 = 2I, I_2 = I$.
- d) $I_1 = 2I, I_2 = 2I$.

