

Important:

Pts

- 1. Answer all questions and problems (No solution = no points).
- 2. Full mark = 20 points as arranged in the above table.
- 3. Give your final answer in the correct units.
- 4. Assume $g = 10 \text{ m/s}^2$.
- 5. Mobiles are **<u>strictly prohibited</u>** during the exam.
- 6. Programmable calculators, which can store equations, are not allowed.
- 7. Cheating incidents will be processed according to the university rules.

GOOD LUCK

Part I: Short Problems (2 points each)

SP1. The vectors \vec{A} and \vec{B} are shown in the figure. Find $\vec{B} - \vec{A}$ in unit vector notation.

$$\vec{A} = -5\sin(37^\circ)\,\hat{i} + 5\cos(37^\circ)\,\hat{j} = -3\hat{i} + 4\hat{j}$$

$$\vec{B} = 4\hat{i}$$

$$\vec{B} - \vec{A} = 7\hat{i} - 4\hat{j}$$

$$\vec{|A|=5}$$

SP2. Given three vectors: $\vec{A} = 2\hat{\imath} + 3\hat{k}$; $\vec{B} = 2\hat{\imath} + 4\hat{j}$; $\vec{C} = 3\hat{\jmath} + \hat{k}$. Find $\vec{A} \cdot (\vec{B} \times \vec{C})$

$$\vec{B} \times \vec{C} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 0 \\ 0 & 3 & 1 \end{bmatrix} = 4\hat{i} - 2\hat{j} + 6\hat{k}$$
$$\vec{A} \cdot (\vec{B} \times \vec{C}) = 8 + 18 = 26$$

SP3. A cat runs along a straight line from point <u>A to point B</u> and then turns <u>back to point C</u>, as shown. The entire motion took 8 *s*. Find the cat's <u>average velocity</u> for the entire trip.

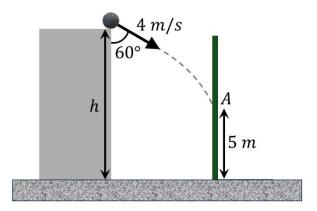
$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_c - x_A}{t_c - t_A}$$

$$\bar{v} = \frac{-10 - 0}{8} = -1.25 \text{ m/s}$$

SP4. A ball is projected from the top of a building with $v_i = 4 m/s$ at an angle of 60° and hits a wall at point **A**, as shown. What is the height of the building (*h*) if the travel time is 2 seconds?

$$\Delta y = v_{y_i} t - \frac{1}{2} g t^2$$

-(h-5) = -2v_i sin 30 - 5(2)²
h = 29 m



SP5. A ball is thrown vertically upward from the ground. Its height after 2 seconds is 6 m above the ground. Find the maximum height of the ball (h_{max}) .

$$y_{f} = y_{i} + v_{y_{i}}t - \frac{1}{2}gt^{2}$$

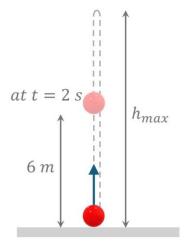
$$6 = 0 + 2v_{y_{i}} - 5(2)^{2}$$

$$v_{y_{i}} = 13 m/s$$

$$v_{f}^{2} = v_{i}^{2} - 2g(y_{f} - y_{i})$$

$$0 = 169 - 20(y_{f} - 0)$$

$$y_{f} = h = 8.45 m$$



Part II: Long Problems (3 points each)

LP1. A particle moves along **the y-axis**. Its position as a function of time is given by $y(t) = 6 + 12t - 5t^2$ where y is in m and t is in s.

a) What is the velocity of the particle at t = 0 s and at t = 2 s.

$$v_y(t) = \frac{dy}{dt} = 12 - 10t$$

$$v_y(0) = 12 - 10(0) = 12 m/s$$

$$v_y(2) = 12 - 10(2) = -8 m/s$$

b) At what time does the particle change its direction?

$$v_y(t) = 0 = 12 - 10t$$

 $t = 1.2 s$

c) Find the distance that the particle covers from t = 0 s to t = 2 s.

$$\Delta y_1 = y(1.2) - y(0) = [6 + 12(1.2) - 5(1.2)^2] - [6 + 12(0) - 5(0)^2] = 7.2 m$$

$$\Delta y_2 = y(2) - y(1.2) = [6 + 12(2) - 5(2)^2] - [6 + 12(1.2) - 5(1.2)^2] = -3.2 m$$

$$d = |\Delta y_1| + |\Delta y_2| = 7.2 + 3.2 = 10.4 m$$

LP2. At t = 0, a particle starts from the **origin** with an initial velocity of $\vec{v}_i = (9\,\hat{j})\,m/s$ and moves in the *x*-*y* plane with a constant acceleration of $\vec{a} = (2\hat{i} - 4\hat{j})\,m/s^2$.

a) Find the time at which the particle reaches x = 15 m.

$$x_{f} = x_{i} + v_{x_{i}}t + \frac{1}{2}a_{x}t^{2}$$

$$15 = 0 + 0 + \frac{1}{2}(2)t^{2}$$

$$t = 3.87 s$$

b) Find the particle's speed at
$$t = 3 s$$
.

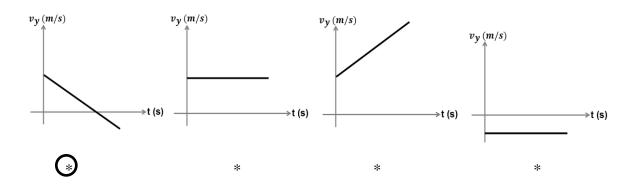
$$\vec{v}_f = \vec{v}_i + \vec{a}t = 9\,\hat{j} + (2\hat{i} - 4\hat{j})(3) = (6\hat{i} - 3\hat{j})$$
$$|v| = \sqrt{(6)^2 + (-3)^2} = 6.7\,m/s$$
OR

$$v_x = v_{x_i}t + a_x t = 0 + 2(3) = 6 m/s$$

$$v_y = v_{y_i}t + a_y t = 9 - 4(3) = -3 m/s$$

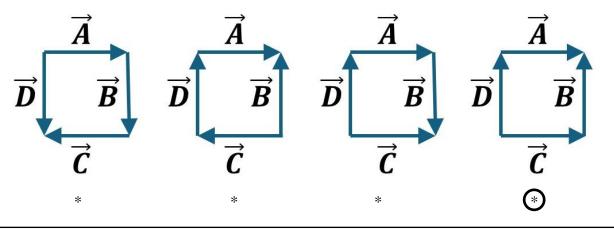
$$|v| = \sqrt{(6)^2 + (-3)^2} = 6.7 m/s$$

c) Which of the following graphs best represents the particle's vertical velocity v_y versus t?

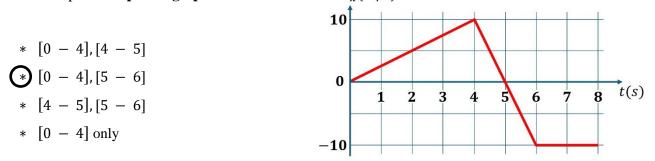


Part III: Questions (Choose the correct answer, one point each)

Q1. Which of the following figures satisfies the relation $\vec{B} = \vec{A} + \vec{D} - \vec{C}$:



Q2. The figure shows the velocity-time graph for a particle moving along the x-axis. During which time intervals is the particle speeding up? $v_x(m/s)$



Q3. A projectile is fired from the ground with an initial velocity (\vec{v}_0) at an angle (θ) with the horizontal, as shown. At point C, what are the signs of $(v_x \text{ and } v_y)$, respectively?



Q4. Two objects are moving with the <u>same constant speed</u> ($v_A = v_B$) in two concentric circles, as shown. If the magnitude of acceleration for object A is a_A and for object B is a_B , then which of the following statements is correct:

