



**Second Midterm Examination
Fall Semester 2023 – 2024**

December 9, 2023

Time: 11:00 AM – 12:30 PM

Name: Student No:

Section No: Serial No:

Instructors: Drs. Alfailakawi, Hadipour, Lajko, Sharma, Vagenas

Fundamental constants

$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2$	(Coulomb constant)
$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)$	(Permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m}/\text{A}$	(Permeability of free space)
$ e = 1.60 \times 10^{-19} \text{ C}$	(Elementary unit of charge)
$N_A = 6.02 \times 10^{23}$	(Avogadro's number)
$g = 9.8 \text{ m/s}^2$	(Acceleration due to gravity)
$m_e = 9.11 \times 10^{-31} \text{ kg}$	(Electron mass)
$m_p = 1.67 \times 10^{-27} \text{ kg}$	(Proton mass)

Prefixes of units

$m = 10^{-3}$	$\mu = 10^{-6}$	$n = 10^{-9}$	$p = 10^{-12}$
$k = 10^3$	$M = 10^6$	$G = 10^9$	$T = 10^{12}$

For use by Instructors only

Problems	1	2	3	4	5	6	7	8	Questions	Total
Marks										

Instructions to the Students:

1. Mobile or other electronic devices are **strictly prohibited** during the exam.
2. Programmable calculators, which can store equations, are not allowed.
3. Cheating incidents will be processed according to the university rules.

PART I: Solve the following problems. Show your solutions in detail.

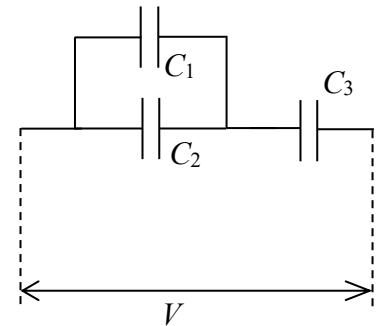
1. In the given network of capacitors with $C_1 = 5 \mu\text{F}$, and $C_2 = 10 \mu\text{F}$, the charge on capacitor C_2 is $Q_2 = 10 \mu\text{C}$. If the stored energy in capacitor C_3 is $11.25 \mu\text{J}$, find the capacitance C_3 . [4 points]

$$V_2 = \frac{Q_2}{C_2} = 1 \text{ V} \Rightarrow V_2 = V_1 = 1 \text{ V} \quad [1]$$

$$Q_1 = C_1 V_1 = 5 \mu\text{C} \quad [1]$$

$$Q_{12} = Q_1 + Q_2 = 15 \mu\text{C} \Rightarrow Q_3 = Q_{12} = 15 \mu\text{C} \quad [1]$$

$$U_3 = \frac{1}{2} \frac{Q_3^2}{C_3} \Rightarrow C_3 = \frac{Q_3^2}{2 \times U_3} \Rightarrow C_3 = 10 \mu\text{F} \quad [1]$$



2. In an air-filled parallel-plate capacitor, the electric energy stored is $60 \mu\text{J}$. The space between the plates of the capacitor is of volume $4 \times 10^{-3} \text{ m}^3$. Find the magnitude of the electric field \vec{E} between the plates. [3 points]

$$u = \frac{U_C}{(\text{volume})} \quad [1]$$

$$u = \frac{60 \times 10^{-6}}{4 \times 10^{-3}} \text{ J/m}^3 \Rightarrow u = 1.5 \times 10^{-2} \text{ J/m}^3 \quad [1]$$

$$u = \frac{1}{2} \epsilon_0 E^2 \Rightarrow E = \sqrt{\frac{2u}{\epsilon_0}} \Rightarrow E = 58,222 \text{ N/C} \quad [1]$$

3. An air-filled parallel-plate capacitor with a surface area A and a plate separation d , has capacitance C_0 . When the capacitor is filled with two dielectric materials with dielectric constants $K_1 = 3$ and K_2 , as shown, the capacitance is $C = 4C_0$. Calculate the dielectric constant K_2 . [5 points]

C_1 and C_2 are in parallel

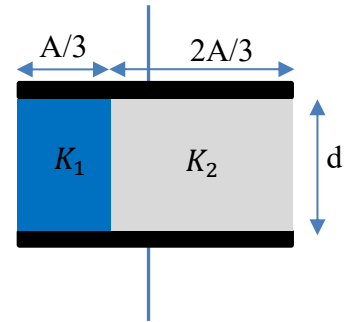
$$C = C_1 + C_2 \quad [1]$$

$$C_1 = K_1 \varepsilon_0 \frac{A/3}{d} = \frac{K_1}{3} \varepsilon_0 \frac{A}{d} = \frac{K_1}{3} C_0 = C_0 \quad [1]$$

$$C_2 = K_2 \varepsilon_0 \frac{2A/3}{d} = \frac{2K_2}{3} \varepsilon_0 \frac{A}{d} = \frac{2K_2}{3} C_0 \quad [1]$$

$$C = C_0 + \frac{2K_2}{3} C_0 \Rightarrow 4C_0 = C_0 + \frac{2K_2}{3} C_0 \quad [1]$$

$$4 = 1 + \frac{2}{3} K_2 \Rightarrow K_2 = 4.5 \quad [1]$$



4. A cylindrical wire of material of resistivity $\rho = 1.72 \times 10^{-8} \Omega \cdot \text{m}$ has length 25 m and radius 0.1 mm. The power dissipated in the wire is 11.6 W. Find the number of electrons that pass through the cross section of the wire in 2 ms. [5 points]

$$R = \rho \frac{L}{A} \Rightarrow R = \rho \frac{L}{\pi r^2} \Rightarrow R = 13.6 \Omega \quad [1]$$

$$P = I^2 R \Rightarrow I = \sqrt{\frac{P}{R}} \quad [1]$$

$$I = 0.92 \text{ A} \quad [1]$$

$$Q = I \Delta t \Rightarrow Q = 1.84 \times 10^{-3} \text{ C} \quad [1]$$

$$Q = N|e| \Rightarrow N = 1.15 \times 10^{16} \text{ electrons} \quad [1]$$

5. The power dissipated in the $20\ \Omega$ resistance is $10.4\ \text{W}$. Find the electric currents I_1 , I_2 , and I_3 .

[3 points]

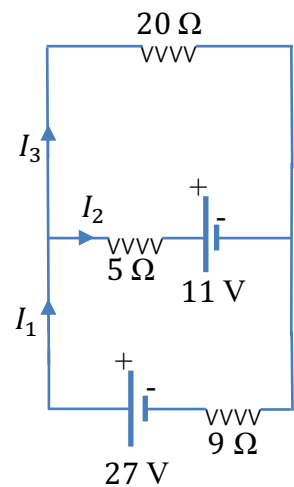
$$P_{20\Omega} = I_3^2(20\ \Omega) \Rightarrow I_3 = 0.72\ \text{A} \quad [1]$$

Big Loop (clockwise)

$$27 - 20I_3 - 9I_1 = 0 \Rightarrow I_1 = 1.4\ \text{A} \quad [1]$$

Upper Loop (clockwise)

$$11 + 5I_2 - 20I_3 = 0 \Rightarrow I_2 = 0.68\ \text{A} \quad [1]$$



6. The capacitor C is initially uncharged and at time $t = 0$ the switch is closed. Find the electric current at time $t_1 = 2\ \text{ms}$ after the switch is closed.

[4 Points]

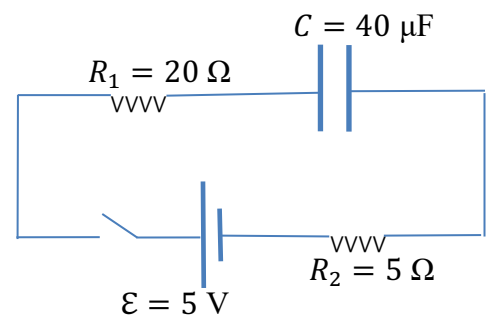
$$\tau = R_{eq}C = (R_1 + R_2)C \Rightarrow \tau = 1\ \text{ms} \quad [1]$$

$$q(t) = C\mathcal{E}(1 - e^{-\frac{t}{\tau}})$$

$$i(t) = \frac{dq(t)}{dt} = \frac{C\mathcal{E}}{\tau} e^{-\frac{t}{\tau}} \quad [2]$$

$$i(t_1) = \frac{C\mathcal{E}}{\tau} e^{-\frac{t_1}{\tau}} \Rightarrow i(t_1) = \frac{200 \times 10^{-6}}{10^{-3}} e^{-\frac{0.002}{0.001}}\ \text{A}$$

$$i(t_1) = 2.71 \times 10^{-2}\ \text{A} \quad [1]$$



7. A particle with charge $q = -3.0 \text{ nC}$ is moving in a magnetic field $\vec{B} = 4.5 \text{ T } \hat{k}$. If the magnetic force exerted on the particle is $\vec{F} = 2.70 \times 10^{-9} \text{ N } (-\hat{i}) + 4.05 \times 10^{-9} \text{ N } \hat{j}$, then calculate the components of the velocity \vec{v} of the particle. **[4 Points]**

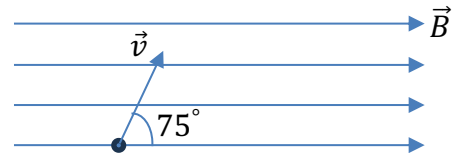
$$\vec{F} = q \vec{v} \times \vec{B} = (-3 \times 10^{-9}) \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ 0 & 0 & 4.5 \end{vmatrix} \quad [1]$$

$$\vec{F} = (-3 \times 10^{-9}) [4.5v_y \hat{i} - 4.5v_x \hat{j}] \quad [1]$$

$$2.70 \times 10^{-9} (-\hat{i}) + 4.05 \times 10^{-9} \hat{j} = 1.35 \times 10^{-8} v_y (-\hat{i}) + 1.35 \times 10^{-8} v_x (-\hat{j}) \quad [1]$$

$$v_x = 0.30 \text{ m/s}, \quad v_y = 0.20 \text{ m/s}, \text{ and } v_z \text{ is indetermined} \quad [1]$$

8. A proton with speed $v = 2 \times 10^5 \text{ m/s}$ enters a uniform magnetic field $\vec{B} = 8.0 \times 10^{-3} \text{ T } \hat{i}$, as shown. Find the pitch of its helical motion. **[3 Points]**



$$v_x = v \cos(75^\circ) \Rightarrow v_x = 51,764 \text{ m/s} \quad [1]$$

$$T = \frac{2\pi m_p}{B|e|} \Rightarrow T = 8.20 \times 10^{-6} \text{ s} \quad [1]$$

$$P = v_x T \Rightarrow P = 0.424 \text{ m} \quad [1]$$

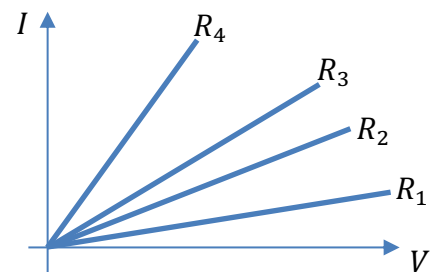
PART II: Conceptual Questions (each carries 1 point). Tick the best answer:

- The capacitance of a capacitor depends only on
 - its potential difference.
 - the capacitor's stored energy.
 - its geometry and the matter between its plates. (ANSWER)
 - its electric charge.

- An air-filled capacitor with plate separation d , area A and capacitance C_0 is charged by a battery V_0 . The battery is disconnected. If a slab of dielectric constant K fully fills the space between the plates of the capacitor, then the potential difference between the plates is
 - V_0 .
 - V_0/K . (ANSWER)
 - KV_0 .
 - zero.

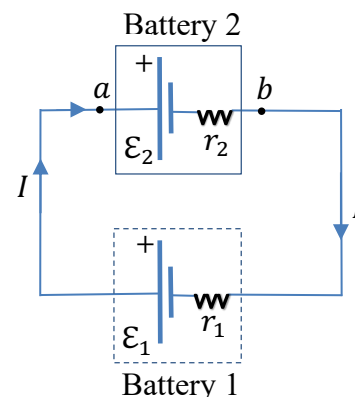
- In the figure below the characteristic curves of four resistances are drawn. Which statement is correct?

- $R_1 > R_2 > R_4 > R_3$
- $R_4 > R_3 > R_2 > R_1$
- $R_4 > R_2 > R_3 > R_1$
- $R_1 > R_2 > R_3 > R_4$ (ANSWER)



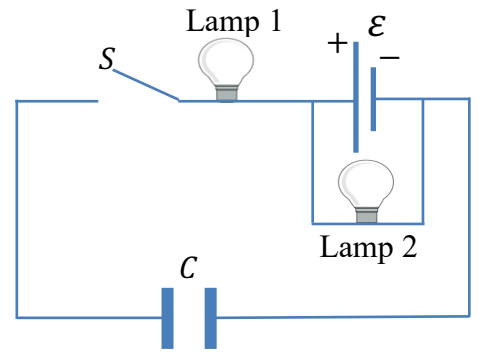
- In the circuit shown, two batteries are connected as shown. The terminal voltage of battery 2 is

- $V_{ab} = \mathcal{E}_1 + Ir_1$.
- $V_{ab} = \mathcal{E}_2 + Ir_2$. (ANSWER)
- $V_{ab} = \mathcal{E}_2 - Ir_2$.
- $V_{ab} = \mathcal{E}_1 + I(r_1 + r_2)$



5. In the RC circuit, the switch S is closed at $t = 0$ s. After a very long time,

- a) lamp 1 and lamp 2 will light.
- b) lamp 1 will not light but lamp 2 will light. (ANSWER)
- c) lamp 1 and lamp 2 will not light.
- d) lamp 1 will light up but lamp 2 will not light.



6. Which statement is correct?

- a) The magnetic field lines can be open or closed.
- b) The magnetic field lines are parallel with the magnetic field \vec{B} . (ANSWER)
- c) The magnetic field lines are perpendicular to the magnetic field \vec{B} .
- d) The magnetic field lines are always perpendicular to the electric field lines.

7. Which statement is **not** equivalent with the Gauss's law for magnetism?

- a) We have never observed magnetic monopoles.
- b) The magnetic field lines have no starting and end points.
- c) The electric charge is conserved in any closed system. (ANSWER)
- d) The magnetic charge does not exist.

8. Three particles enter a uniform magnetic field at point P. Which statement is correct?

- a) Particle 1 is negatively charged and particle 2 is positively charged.
- b) Particle 2 is neutral and particle 3 is positively charged.
- c) Particle 2 is negatively charged and particle 3 is neutral.
- d) Particle 1 is positively charged and particle 2 is neutral. (ANSWER)

