


Second Midterm Examination
Fall Semester 2022 – 2023
December 10, 2022
Time: 11:00 AM – 12:30 PM

Name: Student No:

Section No: Serial No:

Instructors: Drs. Alfrousheh, Al-Failakawi, Farhan, Lajko, Vagenas

Fundamental constants

$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2$	(Coulomb constant)
$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)$	(Permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$	(Permeability of free space)
$ e = 1.60 \times 10^{-19} \text{ C}$	(Elementary unit of charge)
$N_A = 6.02 \times 10^{23}$	(Avogadro's number)
$g = 9.8 \text{ m/s}^2$	(Acceleration due to gravity)
$m_e = 9.11 \times 10^{-31} \text{ kg}$	(Electron mass)
$m_p = 1.67 \times 10^{-27} \text{ kg}$	(Proton mass)

Prefixes of units

$m = 10^{-3}$	$\mu = 10^{-6}$	$n = 10^{-9}$	$p = 10^{-12}$
$k = 10^3$	$M = 10^6$	$G = 10^9$	$T = 10^{12}$

For use by Instructors only

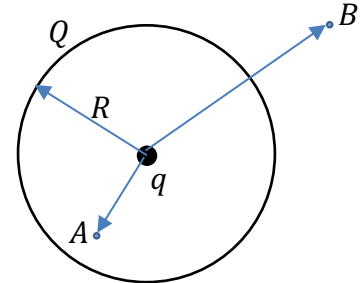
Problems	1	2	3	4	5	6	7	8	Questions	Total
Marks										

Instructions to the Students:

1. Mobile or other electronic devices are **strictly prohibited** during the exam.
2. Programmable calculators, which can store equations, are not allowed.
3. **Cheating incidents will be processed according to the university rules.**

PART I: Solve the following problems. Show your solutions in detail.

1. A thin conducting spherical shell with radius $R = 9$ cm is uniformly charged with charge Q . A point charge $q = -3$ nC is placed at the center of the spherical shell as shown. If the potential is zero at infinity, the potential at point A, $r_A = 7$ cm, is $V_A = 400$ V. Find the potential at point B, $r_B = 13$ cm. **[4 points]**



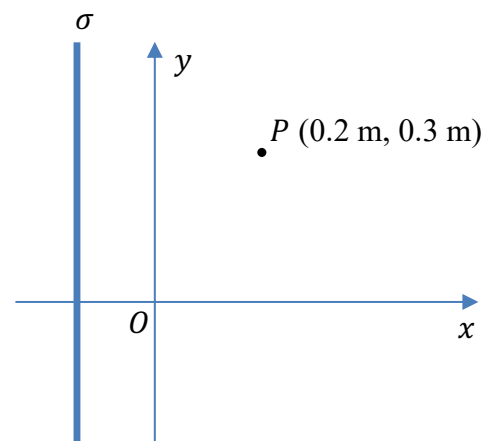
$$V_A = V_q + V_Q = k \frac{q}{0.07} + k \frac{Q}{0.09}$$

$$Q = 7.86 \text{ nC}$$

$$V_B = V_q + V_Q = k \frac{q}{0.13} + k \frac{Q}{0.13}$$

$$V_B = 336.5 \text{ V}$$

2. An infinite sheet of uniform surface charge density σ is placed perpendicular to the x -axis, as shown below. If the potential difference between the origin O of the axes and point P is 18 V, find the surface charge density σ of the infinite sheet. **[5 points]**



$$V_0 - V_P = \int \vec{E} \cdot d\vec{l}$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{i}$$

$$V_0 - V_P = \int \left[\frac{\sigma}{2\epsilon_0} \hat{i} \right] \cdot d\vec{l}$$

$$18 = \frac{\sigma}{2\epsilon_0} (0.2 - 0)$$

$$\sigma = 1.60 \text{ nC/m}^2$$

3. A network of four capacitors with identical capacitance, $C_1 = C_2 = C_3 = C_4 = 9 \text{ nF}$, are connected in a circuit as shown. Calculate the electric charge on capacitor C_3 . **[4 points]**

C_2, C_3, C_4 are in parallel

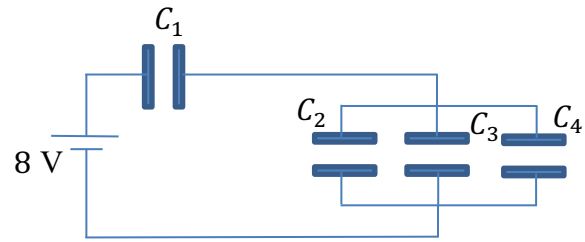
$$C_{234} = C_2 + C_3 + C_4 = 27 \text{ nF}$$

C_{234}, C_1 are in series

$$C_{eq} = \frac{C_1 \times C_{234}}{C_1 + C_{234}} \Rightarrow C_{eq} = 6.75 \text{ nF}$$

$$Q = C_{eq} V \Rightarrow Q = 54 \text{ nC}$$

$$Q_3 = \frac{Q}{3} \Rightarrow Q_3 = 18 \text{ nC}$$



4. Two identical capacitors with capacitances $C_1 = C_2$, are connected to a 20 V battery, as shown in the figure, with charge $Q_0 = 30 \text{ nC}$. Then, while the capacitors remain connected to the battery, a dielectric slab with constant $K = 2$ fully fills the space between the plates of one of the capacitors. Calculate the charge on C_1 after inserting the dielectric material. **[4 points]**

C_1 and C_2 are in series:

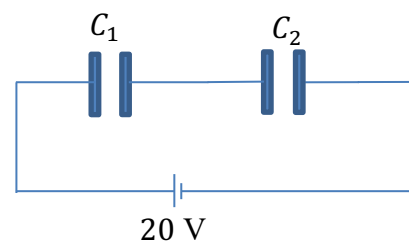
$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{C_1}{2}$$

$$Q_0 = C_{12} V = \frac{C_1}{2} V \Rightarrow C_1 = 3 \text{ nF}$$

After the dielectric is inserted in C_2 :

$$C'_{12} = \frac{C_1 C'_2}{C_1 + C'_2} = \frac{C_1 (2C_2)}{C_1 + 2C_2} = \frac{2}{3} C_1 = 2 \text{ nF}$$

$$Q'_0 = C'_{12} V = 40 \text{ nC}$$



5. An electric current in a wire varies with time as $I(t) = 28.8 \times 10^{-3} \sin(12t)$ with current $I(t)$ in ampere and time t in seconds. Calculate the electric charge that passes through a given cross-section of the wire between time $t = 0$ and $t = \frac{\pi}{4}$ s. **[3 Points]**

$$I(t) = \frac{dQ}{dt} \Rightarrow dQ = I(t)dt \Rightarrow Q = \int_0^{\frac{\pi}{4}} I(t)dt$$

$$Q = 28.8 \times 10^{-3} \int_0^{\frac{\pi}{4}} \sin(12t)dt = \frac{28.8 \times 10^{-3}}{12} \left[-\cos\left(12 \frac{\pi}{4}\right) + \cos(0) \right]$$

$$Q = 4.8 \times 10^{-3} \text{ C}$$

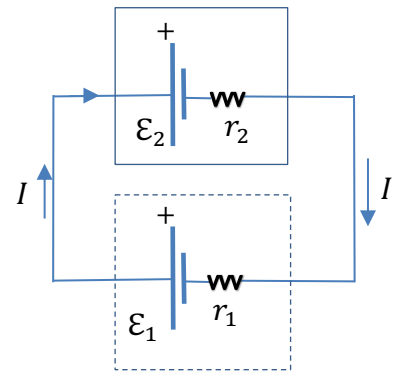
6. In the circuit shown, two batteries are connected as shown. Battery 1 has emf $\mathcal{E}_1 = 15 \text{ V}$ and internal resistance $r_1 = 1.0 \Omega$, and battery 2 has emf \mathcal{E}_2 and internal resistance $r_2 = 1.5 \Omega$. The power dissipated on the internal resistance r_1 is 4 W. Find the emf \mathcal{E}_2 . **[4 Points]**

$$P_{r_1} = I^2 r_1 \Rightarrow I = \sqrt{\frac{P_{r_1}}{r_1}}$$

$$I = 2 \text{ A}$$

$$\mathcal{E}_1 - \mathcal{E}_2 - Ir_1 - Ir_2 = 0$$

$$\mathcal{E}_2 = 10 \text{ V}$$



7. In the circuit shown, calculate the potential difference V_{ab}

[5 Points]

$$R_{up} = \frac{6 \times 6}{6+6} \Omega = 3 \Omega$$

$$R_{down} = \frac{4 \times 4}{4+4} \Omega = 2 \Omega$$

Loop rule

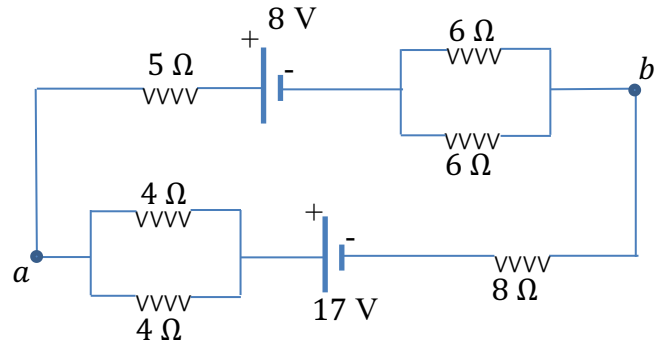
$$17 - 2I - 5I - 8 - 3I - 8I = 0$$

$$I = 0.5 \text{ A}$$

$$V_a - 5 \times 0.5 - 8 - 3 \times 0.5 = V_b$$

$$V_a - V_b = 5 \times 0.5 + 8 + 3 \times 0.5$$

$$V_{ab} = 12 \text{ V}$$



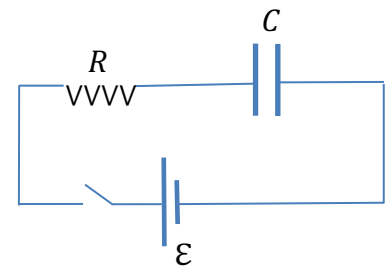
8. In the RC circuit shown below, the time constant is $\tau = 1 \text{ ms}$. At time $t = 0 \text{ s}$, when the switch is closed, the electric current is 3 mA . Find the electric charge at time $t = 2 \text{ ms}$. [3 Points]

$$I_0 = \frac{\mathcal{E}}{R} \Rightarrow \mathcal{E} = I_0 R$$

$$q(t) = \mathcal{E}C \left[1 - e^{-\frac{t}{RC}} \right]$$

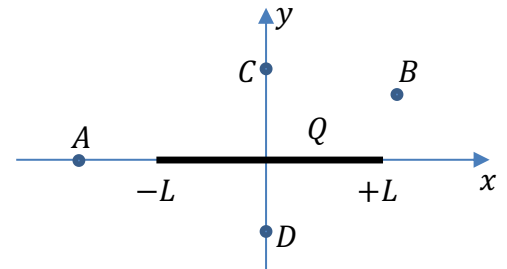
$$q(t) = I_0 RC \left[1 - e^{-\frac{t}{RC}} \right] = I_0 \tau \left[1 - e^{-\frac{t}{\tau}} \right]$$

$$q(t) = 2.59 \mu\text{C}$$



PART II: Conceptual Questions (each carries 1 point). Tick the best answer:

1. A uniformly charged rod of length $2L$ and charge $Q > 0$ lies on the x -axis as shown. If the potential is zero at infinity, which statement is correct?

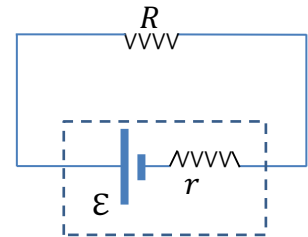


- a) The electric potential at point A is negative.
 - b) The electric potential at point B is positive.**
 - c) The electric potential at point C is negative.
 - d) The electric potential at point C and D is zero.
2. For the equipotential surfaces (EPSs), which statement is correct?
- a) The EPSs of a point charge are infinite planes
 - b) The EPSs of a point charge are infinite lines.
 - c) The EPSs of a point charge are concentric spheres with the point charge at the center.**
 - d) The EPSs of a point charge are of cubic shape.
3. A capacitor is charged by a battery. After disconnecting the battery, the space between the two conductors of the capacitor is fully filled with a dielectric material. The energy stored in the capacitor will
- a) increase.
 - b) decrease.**
 - c) remain the same.
 - d) increase and later decrease.
4. For a network of three identical capacitors $C_1 = C_2 = C_3$, which statement is correct?
- a) If all three are connected in series, the equivalent capacitance has its minimum value.**
 - b) If all three are connected in series, the equivalent capacitance has its maximum value.
 - c) If all three are connected in parallel, the equivalent capacitance has its minimum value.
 - d) If two are in parallel and then in series with the third one, the equivalent capacitance is zero.

5. If the potential difference across a conducting wire is increased, then
- the current density will increase, and the drift speed will decrease.
 - the current density will remain the same and the drift speed will increase.
 - the current density will increase, and the drift speed will increase.**
 - the current density will increase, and the drift speed will remain the same.

6. In the electric circuit below, if the resistance R is increased, then

- the terminal voltage of the battery will increase.**
- the terminal voltage of the battery will remain the same.
- the power dissipated in the internal resistance will increase.
- the power output of the battery will remain the same.



7. The Kirchhoff junction rule is a consequence of

- the conservation of linear momentum.
- the conservation of electric charge.**
- the conservation of angular momentum.
- the conservation of electric energy.

8. In a charging RC circuit, which diagram gives the electric current as a function of time

