



**Final Examination**  
**Fall Semester 2023 – 2024**

**January 02, 2024**  
**Time: 2:00 – 4:00 PM**

Name: ..... Student No: ..... Sec. No: ..... Serial No: .....

Instructors: Drs. Alaa Alfaiakawi, Afshin Hadipour, Peter Lajko, Madan Sharma,  
and Elias Vagenas

**Fundamental constants**

$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2$	(Coulomb constant)
$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)$	(Permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A}$	(Permeability of free space)
$ e  = 1.60 \times 10^{-19} \text{ C}$	(Elementary unit of charge)
$N_A = 6.02 \times 10^{23}$	(Avogadro's number)
$g = 9.8 \text{ m/s}^2$	(Acceleration due to gravity)
$m_e = 9.11 \times 10^{-31} \text{ kg}$	(Electron mass)
$m_p = 1.67 \times 10^{-27} \text{ kg}$	(Proton mass)

**Prefixes of units**

$m = 10^{-3}$	$\mu = 10^{-6}$	$n = 10^{-9}$	$p = 10^{-12}$
$k = 10^3$	$M = 10^6$	$G = 10^9$	$T = 10^{12}$

**For use by Instructors only**

Problems	1	2	3	4	5	6	7	8	9	10	Questions	Total
Marks												

**Instructions to the Students:**

1. Mobile or other electronic devices are **strictly prohibited** during the exam.
2. Programmable calculators, which can store equations, are not allowed.
3. store equations, are not allowed.
4. Cheating incidents will be processed according to the university rules.

**PART I: Solve the following problems. Show your solutions in detail.**

1. Three point charges,  $q_1 = -2\mu\text{C}$ ,  $q_2 = 2\mu\text{C}$ , and  $q_3 = 2\mu\text{C}$ , are placed on the  $xy$ -plane, as shown. Calculate the  $x$  and  $y$  components of the net electric force,  $\vec{F}_3$ , acting on  $q_3$ . **[4 points]**

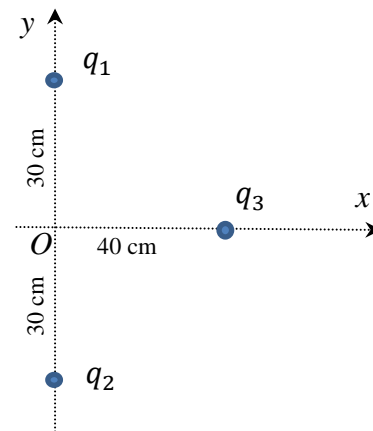
$$r_{13} = r_{23} = \sqrt{(0.3\text{m})^2 + (0.4\text{m})^2} = 0.5\text{ m}$$

$$F_{13,x} = -k \frac{|q_1 q_3|}{r_{13}^2} \cos(\theta); \quad F_{13,y} = k \frac{|q_1 q_3|}{r_{13}^2} \sin(\theta);$$

$$F_{23,x} = k \frac{|q_2 q_3|}{r_{23}^2} \cos(\theta); \quad F_{23,y} = k \frac{|q_2 q_3|}{r_{23}^2} \sin(\theta);$$

$$F_{3,x} = F_{13,x} + F_{23,x} = -k \frac{|q_1 q_3|}{r_{13}^2} \frac{4}{5} + k \frac{|q_2 q_3|}{r_{23}^2} \frac{4}{5} = 0\text{ N}$$

$$F_{3,y} = F_{13,y} + F_{23,y} = k \frac{|q_1 q_3|}{r_{13}^2} \frac{3}{5} + k \frac{|q_2 q_3|}{r_{23}^2} \frac{3}{5} = 0.17\text{ N}$$



2. A spherical shell of inner radius  $a = 8\text{ cm}$  and outer radius  $b = 16\text{ cm}$  has uniform volume charge density  $\rho = -250\text{ nC/m}^3$ , and a point charge  $q = 3\text{ nC}$  is placed at the center of the shell. Determine the magnitude and direction of the net electric field at a distance of  $24\text{ cm}$  from the center. **[4 points]**

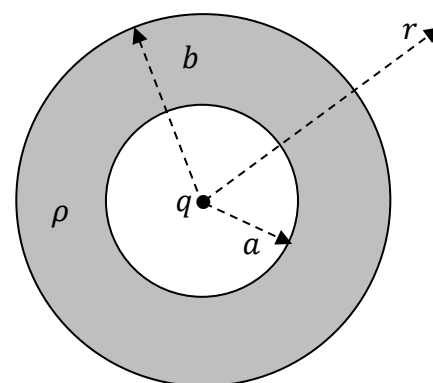
Gauss's Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{encl} = q + \rho \left( \frac{4}{3} \pi (b^3 - a^3) \right)$$

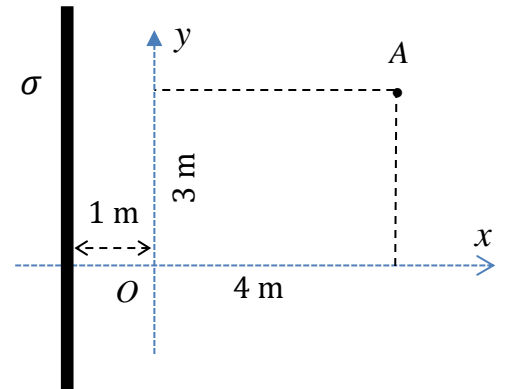
$$E = \frac{Q_{enc}}{4\pi r^2 \epsilon_0} = -118\text{ N/C, inward}$$



3. A very large sheet with uniform surface charge density  $\sigma = 35.4 \text{ nC/m}^2$  is placed perpendicular to the  $x$ -axis, as shown. If a point charge  $q = 4 \text{ nC}$  is moved from point  $A$  to  $O$ , calculate the work done by the electric field. [3 points]

$$W_E = \int_A^O \vec{F} \cdot d\vec{l} = \vec{F} \cdot \Delta\vec{L}$$

$$\vec{F} \cdot \Delta\vec{L} = \frac{\sigma q}{2\epsilon_0} \Delta x = -3.2 \times 10^{-5} \text{ J}$$



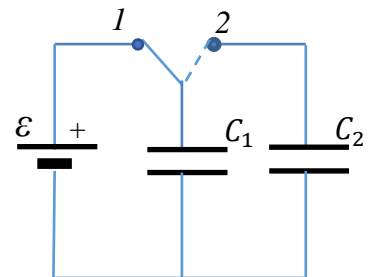
4. A capacitor,  $C_1 = 20 \mu\text{F}$ , is charged by a battery, as shown. Then the switch is moved to position 2 so that capacitor  $C_1$  is connected to uncharged capacitor  $C_2$ . If the initial energy stored in  $C_1$  is  $U_{in} = 90 \mu\text{J}$  and the final energy stored in  $C_1$  is  $U_{fin} = 40 \mu\text{J}$ , determine the value of  $C_2$ . [4 points]

$$U_{in} = \frac{Q_0^2}{2C_1} \Rightarrow Q_0 = \sqrt{2C_1 U_{in}} = 60 \mu\text{C}$$

$$U_{fin} = \frac{Q_1^2}{2C_1} \Rightarrow Q_1 = \sqrt{2C_1 U_{fin}} = 40 \mu\text{C}$$

$$Q_0 = Q_1 + Q_2 \text{ so } Q_2 = Q_0 - Q_1 = 20 \mu\text{C}$$

$$V_1 = V_2 \Rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \Rightarrow C_2 = \frac{C_1 Q_2}{Q_1} = 10 \mu\text{F}$$

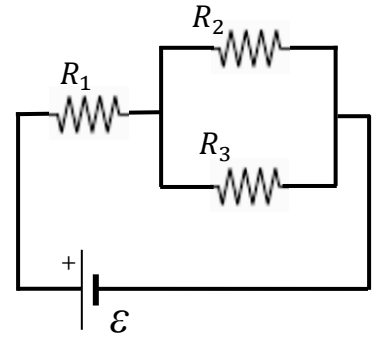


5. When a battery is connected to a network of resistors, as shown, the power dissipated in  $R_2$  is 16 W.  
If  $R_1 = R_2 = R_3 = 4 \Omega$ , determine the value of  $\mathcal{E}$ . **[4 points]**

$$P = I_2^2 R_2 = 16 \text{ W} \Rightarrow I_2 = 2 \text{ A}$$

$$\text{From Junction rule: } I_1 = I_2 + I_3 = 4 \text{ A}$$

$$\text{Loop rule: } \mathcal{E} = I_1 R_1 + I_2 R_2 = 24 \text{ V}$$



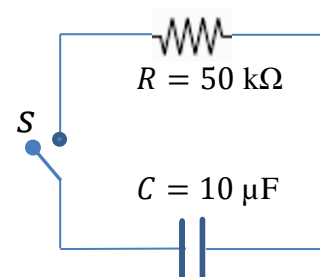
6. In the circuit below, the initial charge on the capacitor is  $Q_0 = 120 \text{ nC}$  and the switch is closed at time  $t = 0 \text{ s}$ . Calculate the time  $t_1$  at which the rate of energy dissipation  $P$  in the resistance drops to 1/9 of its initial value. **[4 points]**

$$P = \frac{V^2}{R} = \frac{Q_0^2}{RC^2} e^{-\frac{2t}{RC}}; P_0 = \frac{Q_0^2}{RC^2}$$

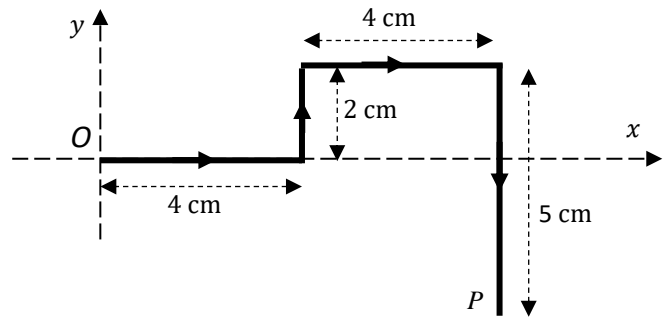
$$\frac{P_0}{9} = P = \frac{Q_0^2}{RC^2} e^{-2t_1/RC}$$

$$\frac{1}{9} = e^{-2t_1/RC} \Rightarrow \frac{1}{3} = e^{-t_1/RC}$$

$$t_1 = -RC \ln\left(\frac{1}{3}\right) = 0.55 \text{ s}$$



7. A current  $I = 7.5 \text{ A}$  flows in a wire from point  $O$  (origin) to point  $P$ , as shown in the figure. Calculate the magnetic force vector acting on the wire due to a uniform magnetic field  $\vec{B} = (3.2 \text{ T})\hat{k}$ . [3 points]



$$\vec{l}_{eff} = (\Delta x)\hat{i} + (\Delta y)\hat{j} = (0.08 \text{ m})\hat{i} + (-0.03 \text{ m})\hat{j}$$

$$\vec{F} = I\vec{l}_{eff} \times \vec{B} = 7.5 \text{ A}[(0.08 \text{ m})\hat{i} - (0.03 \text{ m})\hat{j}] \times [(3.2 \text{ T})\hat{k}]$$

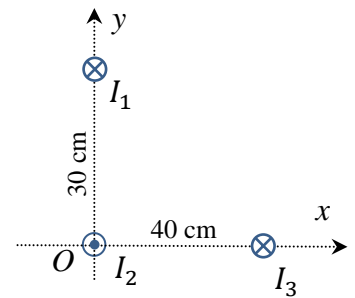
$$\vec{F} = -[(0.72 \text{ N})\hat{i} + (1.92 \text{ N})\hat{j}]$$

8. Three very long wires are perpendicular to the  $xy$ -plane. Each wire carries a current of magnitude  $I = 6 \text{ A}$  in the directions as shown. Calculate the force acting on a 3-m segment of the wire of  $I_2$ . [4 points]

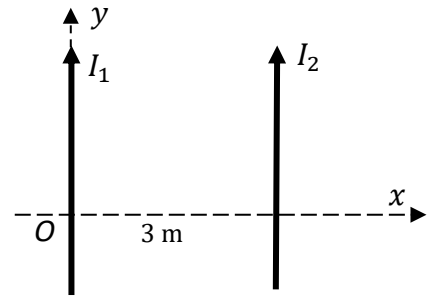
$$F_{12} = \frac{\mu_0 I_1 I_2 L}{2\pi 0.3 \text{ m}} = 7.2 \times 10^{-5} \text{ N}$$

$$F_{32} = \frac{\mu_0 I_1 I_3 L}{2\pi 0.4 \text{ m}} = 5.4 \times 10^{-5} \text{ N}$$

$$\vec{F}_2 = -(5.4 \times 10^{-5} \text{ N})\hat{i} - (7.2 \times 10^{-5} \text{ N})\hat{j}$$



9. Two wires carry currents  $I_1 = 4 \text{ A}$ ,  $I_2 = 2 \text{ A}$  parallel with the  $y$ -axis as shown in the figure. Determine the  $x$  coordinate of the point along the  $x$ -axis where the magnetic field is zero. [3 points]

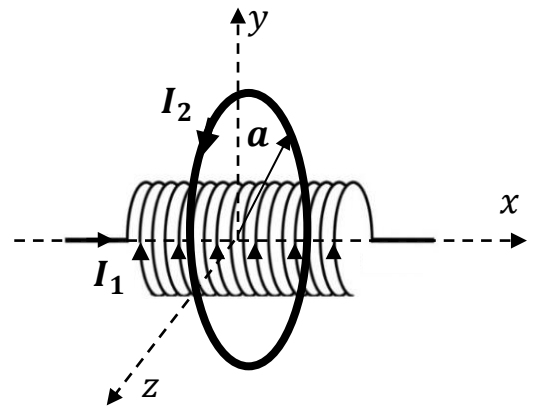


$$\vec{B} = \vec{B}_1 + \vec{B}_2 = 0 \Rightarrow$$

$$B_1 = B_2 \Rightarrow \frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi(3-x)}$$

$$I_1(3-x) = I_2 x \Rightarrow x = 2 \text{ m}$$

10. A long solenoid with its center at the origin has 1500 turns per meter and carries a current  $I_1 = 40 \text{ mA}$ . A ring with radius  $a$  around the solenoid, with its center at the origin, carries a current  $I_2 = 6.0 \text{ A}$  in the  $yz$ -plane. Find the radius of the ring if the net magnetic field at the origin is  $\vec{B} = 0$ . [3 points]



$$\vec{B}_1 = \mu_0 n I_1 (-\hat{i})$$

$$\vec{B}_2 = \frac{\mu_0 I_2}{2a} (+\hat{i})$$

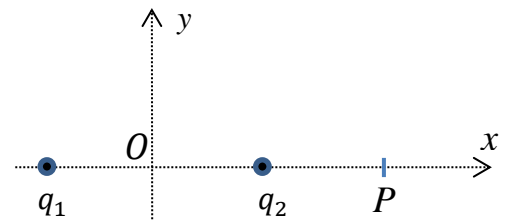
$$\vec{B} = \vec{B}_1 + \vec{B}_2 = 0 \rightarrow \mu_0 n I_1 = \frac{\mu_0 I_2}{2a} \rightarrow a = \frac{I_2}{2n I_1}$$

$$a = \frac{6}{2 \times 1500 \times 0.04} = 5 \text{ cm}$$

**PART II: Conceptual Questions (each carries 1 point). Tick the best answer:**

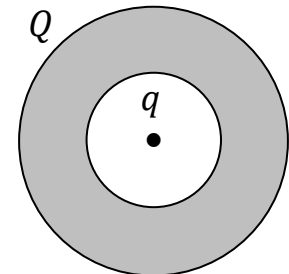
1. Two point charges  $q_1$  and  $q_2$  are fixed on the  $x$ -axis as shown. If the net electric field  $\vec{E}$  is zero at point  $P$ , which statement can be correct for the charges?

- a)  $q_1 = -q_2$ .
- b)  $q_1 < 0$  and  $q_2 < 0$ .
- c)  $q_1 > 0$  and  $q_2 > 0$ .
- d)  $q_1 < 0$  and  $q_2 > 0$ . ←



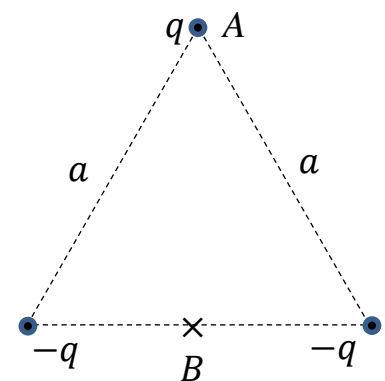
2. A conducting spherical shell has a net charge  $Q$  and a point charge  $q = 5 \text{ nC}$  is fixed at the center of its cavity, as shown. If the net charge on the outer surface of the shell is  $10 \text{ nC}$ , the value of  $Q$  is

- a)  $5 \text{ nC}$ . ←
- b)  $10 \text{ nC}$ .
- c)  $15 \text{ nC}$ .
- d)  $0 \text{ nC}$ .



3. A point charge  $q$  is released from rest at point  $A$  and it moves to point  $B$  shown in the figure. Which statement is true for the potential and kinetic energy of charge  $q$ ?

- a)  $\Delta U > 0$ .
- b)  $\Delta U > \Delta K$ .
- c)  $\Delta U < 0$ . ←
- d)  $\Delta U = \Delta K$ .



4. An air-filled parallel-plate capacitor has capacitance  $C$ . Then the separation between the plates is reduced by half and the area of the plates is doubled, so its capacitance

- a) remains  $C$ .
- b) increases to  $2C$ .
- c) decreases  $C/2$ .
- d) increases to  $4C$ . ←

5. Voltmeters are always connected parallel with the circuit element and ammeters are always connected in series because

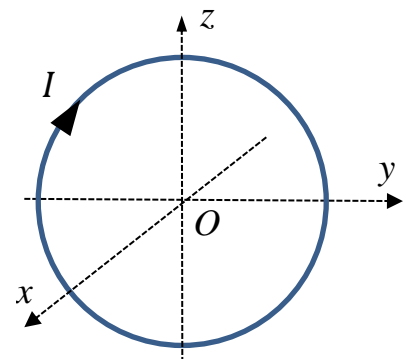
- a) voltmeters have zero resistance and ammeters have infinite large resistance.
- b) voltmeters have infinite large resistance and ammeters have zero resistance. ←
- c) voltmeters have infinite large capacitance and ammeters have infinite large resistance.
- d) voltmeters have zero capacitance and ammeters have infinite large capacitance.

6. If a charged particle moves in a region of uniform magnetic field with a velocity perpendicular to the magnetic field, the path of the particle is a

- a) straight line path.
- b) helical path.
- c) parabolic path.
- d) circular path. ←

7. A circular loop of current is centered at the origin in the  $yz$ -plane as shown. The direction of magnetic field along the  $x$ -axis

- a) is  $\hat{i}$  if  $x < 0$ , and  $-\hat{i}$  if  $x > 0$ .
- b) is  $-\hat{i}$  if  $x < 0$ , and  $\hat{i}$  if  $x > 0$ .
- c) is  $-\hat{i}$  if  $x < 0$ , and  $-\hat{i}$  if  $x > 0$ . ←
- d) is  $\hat{i}$  if  $x < 0$ , and  $\hat{i}$  if  $x > 0$ .



8. The figure shows three closed curves together with their orientation and two identical magnitude of currents perpendicular to the plane of curves. For which curve is the value of the curve integral  $\oint \vec{B} d\vec{l}$  is  $-\mu_0 I$ ?

- a) Curve A.
- b) Curve B. ←
- c) Curve C.
- d) None of the curves.

