

# For Instructors use only

Grades:

#	SP1	SP2	SP3	SP4	SP5	SP6	SP7	LP1	LP2	LP3	Q1	02	23	Q4	Total
	3	3	3	3	3	3	3	5	5	5	1	$\nabla \mathcal{C}$	1	1	40
Pts															
										$\sim$	$\langle \mathcal{O} \rangle$				

## Important:



- 1. Answer all questions and problem (No solution = no points).
- 2. Full mark = 40 points as arranged in the above table.
- 3. Give your final answer in the correct units.
- 4. Assume  $g = 10 \text{ m/s}^2$ .
- 5. Mobiles are **<u>strictly prohibited</u>** during the exam.
- 6. Programmable calculators, which can store equations, are not allowed.
- 7. Cheating incidents will be processed according to the university rules.

## GOOD LUCK

#### Part I: Short Problems (3 points each)

**SP1**. A 1.5 kg object moves in the *xy* plane with a velocity of  $\vec{v} = (4\hat{\iota} - 3\hat{j}) m/s$ . Calculate the angular momentum of the object relative to the origin in unit vector notation, given that its position vector is  $\vec{r} = (5\hat{\iota} + 2\hat{j}) m$ .

$$\vec{L} = \vec{r} \times \vec{p} = m \, \vec{r} \times \vec{v}$$
$$\vec{r} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 2 & 0 \\ 4 & -3 & 0 \end{vmatrix}$$
$$= 0i + 0j + [5(-3) - (2)(4)] \, \hat{k}$$
$$= -23 \, \hat{k} \, m^2 / s$$
$$\vec{L} = m \, \vec{r} \times \vec{v} = 34.5 \, \hat{k} \, kg \cdot m^2 / s$$

**SP2.** Two forces act on a block of mass *m*. The forces are:  $F_1 = 8 N$  along the positive x-axis and  $F_2 = 12 N$  in a direction 53° north of east. Find the mass *m* if the magnitude of its acceleration is  $6 m/s^2$ .

$$\sum F_x = 8 + 12\cos 53^\circ = 15.2 N$$
  

$$\sum F_y = 12\sin 53^\circ = 9.6 N$$
  

$$\left|\sum \vec{F}\right| = \sqrt{(15.2)^2 + (9.6)^2} = 18 N$$
  

$$\left|\sum \vec{F}\right| = ma \Rightarrow m = \frac{\left|\sum \vec{F}\right|}{a} = \frac{18}{6} = 3 kg$$



4

**SP3.** A2 kg object starts moving at t = 0 s with an initial velocity of 3 m/s along the +x-axis. If the net force exerted on the object varies with time as shown in the figure, use the impulse momentum theorem to find the speed of the object at t = 8 s.

$$J = \Delta p = Area = 6 + 24 + 6 = 36 \, kg \cdot m/s$$

$$m(v_f - v_i) = 36 \, kg \cdot m/s$$

$$2$$

$$v_f = v_i + \frac{\Delta p}{m} = 3 + \frac{36}{2} = 21 \, m/s$$



SP4. A 3 kg stone initially moving with velocity  $\vec{v} = (5\hat{i} + 4\hat{j} - 8\hat{k}) m/s$ , it explodes into two pieces. After the explosion, the first piece  $(m_1 = 2 kg)$  has a velocity of  $\vec{v}_1 = (7\hat{i} + 2\hat{j} - 14\hat{k}) m/s$ . Find the velocity of the second piece  $(m_2 = 1 kg)$  after the explosion, in unit vector notation.

$$\begin{split} M\vec{v}_i &= m_1\vec{v}_1 + m_2\vec{v}_2\\ 3\big(5\hat{\imath} + 4\hat{\jmath} - 8\hat{k}\big) &= 2\left(7\hat{\imath} + 2\hat{\jmath} - 14\hat{k}\right) + 1(\vec{v}_2)\\ \vec{v}_2 &= \big(15\hat{\imath} + 12\hat{\jmath} - 24\hat{k}\big) - \big(14\hat{\imath} + 4\hat{\jmath} - 28\hat{k}\big) = \big(\hat{\imath} + 8\hat{\jmath} + 4\hat{k}\big)\,m/s \end{split}$$

**SP5.** A disk of radius 0.2 *m* is rotating with **constant angular acceleration** around an axis passing through its center, as shown. It starts rotating at t = 0 s with angular velocity of 12 rad/s. After making 20 *revolutions*, its angular velocity becomes 40 rad/s. Find the tangential acceleration (*in m/s*<sup>2</sup>) of point *p* on the disk.

 $\begin{aligned} \Delta\theta &= 20(2\pi) = 125.6 \, rad \\ \omega_f^2 &= \omega_i^2 + 2\alpha \, \Delta\theta \\ 40^2 &= 12^2 + 2(\alpha)(125.6) \\ \alpha &= 5.8 \, rad/s^2 \\ a_{tan} &= R\alpha = 0.2(5.8) = 1.16 \, m/s^2 \end{aligned}$ 



**SP6**. A 3 kg block **rests on a rough horizontal surface**. A variable force  $F(x) = 6x^2 + 2$ , where F(x) is measured in *N* and *x* is measured in *m*, starts acting on the block. The block starts from rest at the origin and achieves a speed of 8 m/s when reaching the position x = 4 m. Find the work done on the block <u>by</u> friction during this motion.



$$w_{F_x} = \int_{x_i}^{x_f} F_x \, dx = \int_0^4 (6x^2 + 2) dx = 2x^3 + 2x]_0^4 = 136 J$$
  
$$\Delta K = \frac{1}{2} m (v_f^2 - v_i^2) = \frac{1}{2} (3)(8^2 - 0) = 96 J$$
  
$$w_{F_x} + w_{f_k} = \Delta K \implies w_{f_k} = \Delta K - w_{F_x} = 96 - 136 = -40 J$$

**SP7**. You throw a stone straight up with an initial speed of 15 m/s. It passes point A on the way up at a height of 7 m. What is the time required for the stone to travel from point A to point B?

 $\Delta y = v_{yi}t - \frac{1}{2}gt^2$   $7 = (15)t - \frac{1}{2}(10)t^2 \implies t_1 = 0.58 \text{ s}, t_2 = 2.42 \text{ s}$   $\Delta t = t_2 - t_1 = 1.84 \text{ s}$ 

 $-9.2 = 9.2 - 10t \Rightarrow t = 1.84 s$ 

## OR

From the ground to point A  $v_{y_f}^2 = v_{y_i}^2 - 2g\Delta y = 15^2 - 2(10)(7) = 85 \Rightarrow v_{y_f} = \pm 9.2 \text{ m/s}$ From A to B  $v_{y_f} = v_{y_i} - gt$ 



### Part II: Long Problems (5 points each)

**LP1.** Block 1 ( $m_1 = 4 kg$ ) slides **from rest** along a **frictionless** ramp from a height of 5 *m*, and block 2 ( $m_2 = 6 kg$ ) rests on a frictionless horizontal surface, as shown. The two blocks collide at the bottom of the ramp and <u>stick together</u>, then they slide into a rough region ( $\mu_k = 0.25$ ) and come to <u>a stop</u> in distance *d*.

#### (a) Find the speed of block 1 just before the collision.

$$U_i + K_i = U_f + K_f$$
$$m_1gh + 0 = 0 + \frac{1}{2}m_1v_1^2$$
$$v_1 = \sqrt{2gh} = 10 \text{ m/s}$$



(b) Find the speed of the combined blocks immediately after the collision.

$$m_1 v_1 + 0 = (m_1 + m_2) v_2$$
  
(4)(10) = (4 + 6) $v_2 \Longrightarrow v_2 = 4 m/s$ 

#### (c) What is the distance *d*?

$$E_f - E_i = W_{f_k}$$
  

$$0 - \frac{1}{2}(m_1 + m_2)v_2^2 = -\mu_k(m_1 + m_2)gd$$
  

$$d = \frac{v_2^2}{2\mu_k g} = \frac{(4)^2}{2(0.25)(10)} = 3.2 m$$

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**LP2.** An object consists of two small masses ( $m_1 = 0.2 kg$  and  $m_2 = 0.3 kg$ ) joined by **a massless** rigid rod of length 0.8 m, as shown. The object is pivoted about an axis through point O so that it can rotate in a vertical circle. The object starts **from rest** from the horizontal position (position A).

(a) Find the moment of inertia of the object about the rotation axis (point O).

$$I = m_1 r_1^2 + m_2 r_2^2 = 0.2(0.4^2) + 0.3(0.8^2)$$
$$= 0.224 \ kg \cdot m^2$$



(b) Find the angular speed of the object <u>at the vertical position (position B).</u>

$$E_{i} = E_{f}$$

$$m_{1}gy_{1_{i}} + m_{2}gy_{2_{i}} = \frac{1}{2}I\omega_{f}^{2}$$

$$0.2(10)(0.4) + 0.3(10)(0.8) = \frac{1}{2}(0.224)\omega_{f}^{2} \Rightarrow \omega_{f} = 5.35 \, rad/s$$

(c) If  $v_1$  is the speed of  $m_1$  and  $v_2$  is the speed of  $m_2$  at the vertical position, then:

\* 
$$v_1 = v_2$$
 \*  $v_1 > v_2$   $\textcircled{O} v_1 < v_2$ 

(d) As the object rotates from the horizontal to the vertical position, the angular acceleration will

\* increase

Output decrease

\* remain constant

**LP3.** Two blocks  $(m_1 = 2 \ kg \ and \ m_2)$  are connected by a massless string over a pulley  $(R = 0.25 \ m, I = 0.05 \ kg \cdot m^2)$ , as shown. Block 1 slides to the right on **a frictionless horizontal surface** with acceleration of  $a = 4 \ m/s^2$  and block 2 slides down **a frictionless incline**.

(a) Find the angular acceleration of the pulley.

$$\alpha = \frac{a_{tan}}{R} = \frac{4}{0.25} = 16 \, rad/s^2$$



### (b) Find the tensions $(T_1 \text{ and } T_2)$ in the string.

#### To find $T_1$

 $T_1 = m_1 a = 8 N$ 

### To find $T_2$

$$T_2 R - T_1 R = I \alpha \implies T_2 = \frac{I \alpha + T_1 R}{R} = \frac{0.05(16) + 8(0.25)}{0.25} = 11.2 N$$

(c) Find the mass of block 2.

$$m_2g\sin(30^\circ) - T_2 = m_2a \implies m_2 = \frac{T_2}{g\sin(60^\circ) - a} = 2.4 \ kg$$

#### Part III: Questions (Choose the correct answer, one point each)

Q1. When a man exerts a force of magnitude (F) to push a box of mass (m) on a rough horizontal surface, the box, exerts a force of magnitude (P) on the man's hand. If the man and the box move with a constant acceleration of magnitude (a), then

\* 
$$F = P + ma - f_k$$
  
\*  $F = P + ma$   
\*  $F = P + ma + f_k$   
 $\bigcirc$   $F = P$ 



**Q2.** A disk is rotating <u>at a constant angular acceleration</u> about a fixed axis. Initially, it rotates clockwise, then it changes its direction of rotation to be counterclockwise. The graph which describes the object's angular velocity ( $\omega$ ) is:



Q3. Three horizontal forces of equal magnitude are applied to a massless rod of length *L*, which is free to rotate about a vertical axis passing through point O, as shown. The magnitude of the net torque on the rod equals



Q4. Two blocks  $(m_1 > m_2)$  slide from rest on two frictionless slides, as shown. Both slides have the same height *h*. At the slides' end, the speed of block 1 is  $v_1$  and the speed of block 2 is  $v_2$ . Which of the following is correct?

↑ h



\* 
$$\nu_1 < \nu_2$$

(\*) 
$$v_1 = v_2$$

\* The relation between  $v_1$  and  $v_2$  depends on the curvature of the curved slide.

h