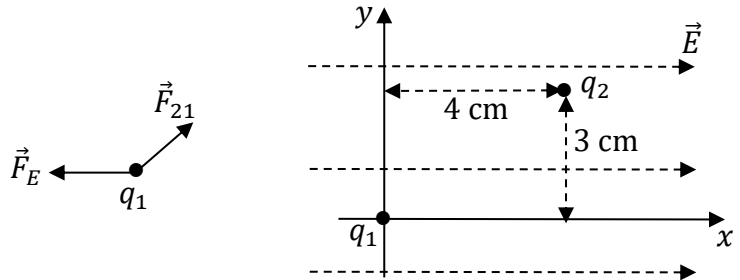




**Part I. Solve the following problems. Show your solutions in detail.**

1. Two point charges  $q_1 = -5.0 \mu\text{C}$  and  $q_2 = 6.0 \mu\text{C}$  are placed in a uniform electric field of  $\vec{E} = (2.0 \times 10^7 \text{ N/C})\hat{i}$  as shown. Find the net electric force  $\vec{F}$  on charge  $q_1$ . [4 points]



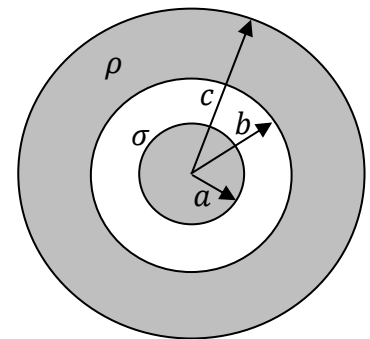
$$\vec{F}_{21} = k \frac{|q_1 q_2|}{r^2} (\cos\theta \hat{i} + \sin\theta \hat{j})$$

$$= (86.4 \text{ N})\hat{i} + (64.8 \text{ N})\hat{j}$$

$$\vec{F}_E = q_1 \vec{E} = -(100 \text{ N})\hat{i}$$

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_E = -(13.6 \text{ N})\hat{i} + (64.8 \text{ N})\hat{j}$$

2. A charged conducting sphere of radius  $a = 4.0 \text{ cm}$  and surface charge density of  $\sigma = -2.0 \mu\text{C}/\text{m}^2$  is concentric with a uniformly charged spherical shell of inner radius  $b = 6.0 \text{ cm}$  and outer radius  $c = 9.0 \text{ cm}$ . The volume charge density of the shell is  $\rho = 8.0 \mu\text{C}/\text{m}^3$ . Find the magnitude and direction of the net electric field at a radial distance of  $12.0 \text{ cm}$  from the center. [4 points]



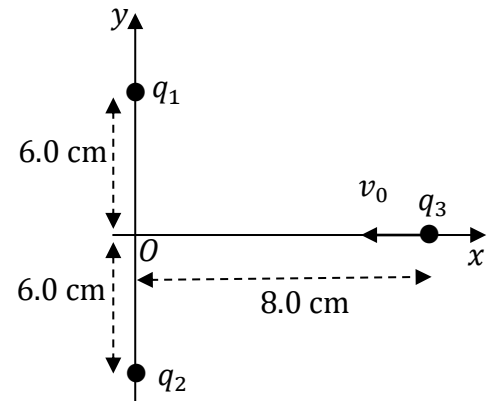
$$q_1 = \sigma \cdot 4\pi a^2 = -40.2 \text{ nC}$$

$$q_2 = \rho \cdot \frac{4\pi}{3} (c^3 - b^3) = 17.2 \text{ nC}$$

$$E = k \frac{|q_1 + q_2|}{r^2} = 14.4 \times 10^3 \text{ N/C}$$

Inward

3. Two point charges  $q_1 = q_2 = q$  are fixed on the  $y$ -axis and a point charge  $q_3 = 4.0 \mu\text{C}$  with a mass of  $m_3 = 3 \times 10^{-6} \text{ kg}$  is on the  $x$ -axis moving towards the origin as shown. The speed of  $q_3$  is  $v_0 = 2 \times 10^3 \text{ m/s}$  at a distance of  $8.0 \text{ cm}$  from the origin. If  $q_3$  stops momentarily at a distance of  $3.0 \text{ cm}$  from the origin, find the value of  $q$ . **[4 Points]**



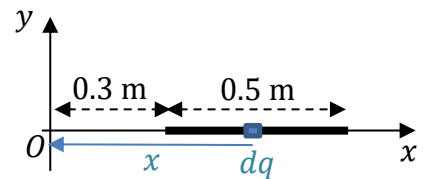
$$2k \frac{qq_3}{r_1} + \frac{1}{2} m_3 v_0^2 = 2k \frac{qq_3}{r_2} + 0$$

$$r_1 = 10 \text{ cm} \quad \text{and} \quad r_2 = 6.7 \text{ cm}$$

$$7.2 \times 10^5 q + 6 = 1.08 \times 10^6 q$$

$$q = 16.9 \mu\text{C}$$

4. A uniformly charged line of length  $0.5 \text{ m}$  and net charge  $Q = 6.0 \text{ nC}$  is along the  $x$ -axis as shown. Find the electric potential of the line at the origin. **[4 points]**



$$dV = k \frac{dq}{x} = k \frac{\lambda dx}{x}$$

$$V = k \frac{Q}{L} \int_{0.3}^{0.8} \frac{dx}{x}$$

$$V = k \frac{Q}{L} (\ln x) \Big|_{0.3}^{0.8}$$

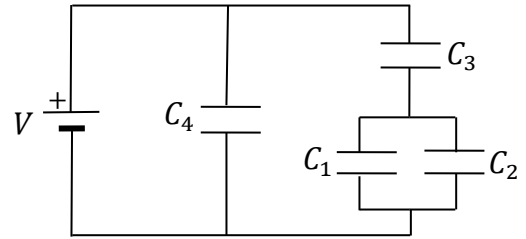
$$V = 106 \text{ V}$$

5. In the capacitor network below  $C_1 = C_2 = 6.0 \mu\text{F}$ ,  $C_3 = C_4 = 12.0 \mu\text{F}$  and the charge on capacitor  $C_4$  is  $Q_4 = 24 \mu\text{C}$ . Find the charge on capacitor  $C_3$ . **[4 points]**

$$C_{12} = 12 \mu\text{F}, \quad C_{123} = 6 \mu\text{F}$$

$$V_4 = V_{123} = \frac{Q_4}{C_4} = 2 \text{ V}$$

$$Q_{123} = Q_3 = C_{123}V_{123} = 12 \mu\text{C}$$



6. An electric field of magnitude  $0.86 \text{ V/m}$  is applied in a conducting wire of resistivity  $1.72 \times 10^{-8} \Omega \cdot \text{m}$  and diameter  $0.2 \text{ mm}$ . How much charge goes through a cross-section of the wire in  $0.5 \text{ min}$ ? **[3 points]**

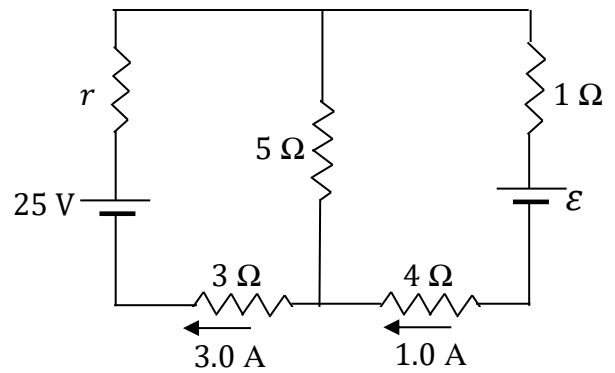
$$J = \frac{E}{\rho} = 5 \times 10^7 \text{ A/m}^2$$

$$I = JA = J\pi\left(\frac{D}{2}\right)^2 = 1.57 \text{ A}$$

$$Q = It = 47.1 \text{ C}$$

7. In the circuit shown find the emf  $\mathcal{E}$  and resistance  $r$ .

[5 points]



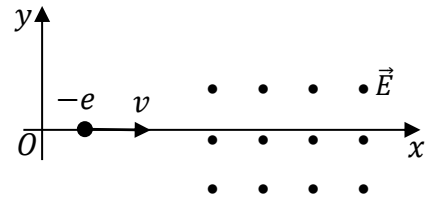
Junction:  $I = 3 - 1 = 2 \text{ A}$

Left loop:  $-3 \times 3 + 25 - 3r - 5 \times 2 = 0 \rightarrow r = 2 \Omega$

Right loop:  $-4 \times 1 + 5 \times 2 - 1 \times 1 - \mathcal{E} = 0 \rightarrow \mathcal{E} = 5 \text{ V}$

8. An electron is accelerated by a potential difference of 80 V, then it goes through a region of crossed electric and magnetic fields with  $\vec{E} = (1000 \text{ N/C})\hat{k}$  as shown. Find a magnetic field  $\vec{B}$  that keeps the electron on a straight path.

[3 points]



$$eV = \frac{1}{2}mv^2 \rightarrow v = 5.3 \times 10^6 \text{ m/s}$$

$$\vec{E} = -\vec{v} \times \vec{B} \rightarrow E\hat{k} = -v\hat{i} \times \vec{B} \rightarrow \vec{B} = B_x\hat{i} + B_y(-\hat{j})$$

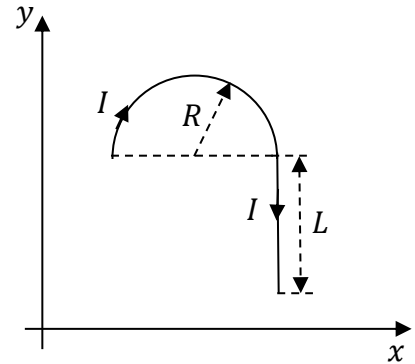
$$B_y = \frac{E}{v} = 1.9 \times 10^{-4} \text{ T}$$

$B_x$  is arbitrary and can't be determined.

9. A wire carrying a current of  $I = 0.5 \text{ A}$  is made of a semicircle of radius  $R = 0.5 \text{ m}$  and a straight section of length  $L = 0.8 \text{ m}$  as shown. The wire is in a magnetic field of  $\vec{B} = (0.4 \text{ T})\hat{i} - (0.6 \text{ T})\hat{j}$ . Calculate the magnetic force  $\vec{F}$  on the wire. [4 points]

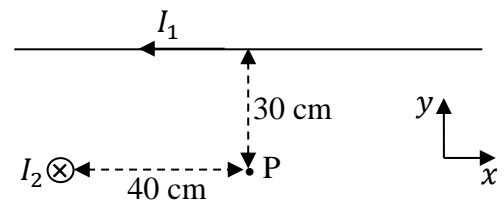
$$\vec{l} = 2R\hat{i} - L\hat{j} = 1.0\hat{i} - 0.8\hat{j} \text{ m}$$

$$\begin{aligned}\vec{F} &= I\vec{l} \times \vec{B} = 0.5(1.0\hat{i} - 0.8\hat{j}) \times (0.6\hat{i} - 0.4\hat{j}) \\ &= -0.3\hat{k} + 0.16\hat{k} = (-0.14 \text{ N})\hat{k}\end{aligned}$$



10. Two infinitely long wires carry currents  $I_1 = 15 \text{ A}$  and  $I_2 = 8 \text{ A}$  as shown. Find the net magnetic field  $\vec{B}$  at point  $P$ . [3 Points]

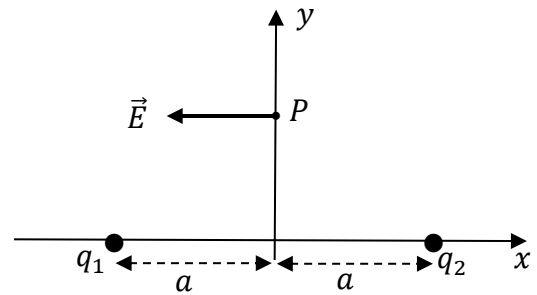
$$\begin{aligned}\vec{B} &= \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I_1}{2\pi r_1} \hat{k} - \frac{\mu_0 I_2}{2\pi r_2} \hat{j} \\ &= -(4 \times 10^{-6} \text{ T})\hat{j} + (1 \times 10^{-5} \text{ T})\hat{k}\end{aligned}$$



**PART II : Conceptual Questions (each carries 1 point). Tick the best answer.**

1. Two point charges  $q_1$  and  $q_2$  of the same magnitude are located on the  $x$ -axis. The net electric field  $\vec{E}$  at point  $P$  is shown on the figure. Which statement is correct?

- a)  $q_1 > 0$  and  $q_2 < 0$ .
- b)  $q_1 > 0$  and  $q_2 > 0$ .
- c)  $q_1 < 0$  and  $q_2 < 0$ .
- d)  $q_1 < 0$  and  $q_2 > 0$ .

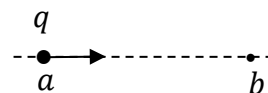


2. A point charge  $q$  is placed inside the cavity of a charged conductor. The charge on the inner and outer surfaces of the conductor is  $Q_{in}$  and  $Q_{out}$ , respectively. The electric field at a point outside the conductor is given by

- a)  $\vec{E} = k \frac{q+Q_{in}}{r^2} \hat{r}$ .
- b)  $\vec{E} = k \frac{q+Q_{out}}{r^2} \hat{r}$ .
- c)  $\vec{E} = k \frac{q}{r^2} \hat{r}$ .
- d)  $\vec{E} = k \frac{Q_{out}}{r^2} \hat{r}$ .

3. A point charge  $q$  that is released from rest at point  $a$  moves to point  $b$ . The change in the electric potential energy of the point charge

- a) **is negative.**
- b) is positive.
- c) is zero.
- d) can be any of the above.

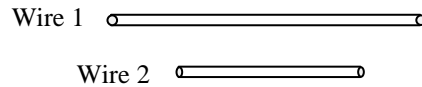


4. An air-filled parallel-plate capacitor is charged and then disconnected from the battery. If the plate separation of this capacitor is increased now,

- a) the capacitor voltage decreases.
- b) the capacitor voltage remains constant.
- c) the energy stored in the capacitor remains constant.
- d) **the energy stored in the capacitor increases.**

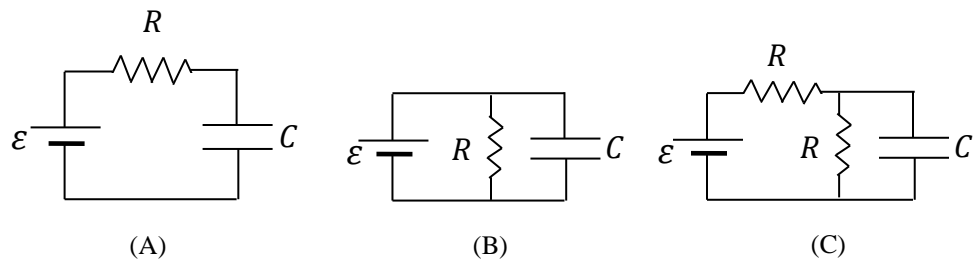
5. Two wires are made of the same material and have equal diameters, but different lengths as shown. If the same potential difference is applied across each wire, the power dissipation

- a) is greater in wire 1.
- b) is greater in wire 2.**
- c) is the same in the two wires.
- d) does not depend on the potential difference.



6. If  $Q_A$ ,  $Q_B$  and  $Q_C$  are the final charges on capacitors in circuits A, B and C respectively, which statement is correct

- a)  $Q_A < Q_B < Q_C$ .
- b)  $Q_C < Q_A = Q_B$ .**
- c)  $Q_C > Q_A = Q_B$ .
- d)  $Q_A = Q_B = Q_C$ .



7. Two charged particles enter a uniform magnetic field. One particle goes on a circular motion and the other on a helical motion. The work done by the magnetic field on these particles

- a) is zero only in circular motion.
- b) is zero only in helical motion.
- c) is zero in both motions.**
- d) is not zero in circular or helical motion.

8. A long straight wire carries a current  $I$  and a positively charged particle  $q$  is launched parallel to the wire as shown. The direction of the magnetic force acting on the particle at this moment is

- a)  $\hat{k}$ .
- b)  $-\hat{j}$ .**
- c)  $-\hat{i}$ .
- d)  $\hat{j}$ .

